EQUIVALENT CIRCUIT MODELS FOR COAXIAL LINE
OFFSET OPEN-CIRCUITS

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1. SUMMARY

Offset open-circuits (O/Cs) have been investigated at NPL (Malvern) as primary impedance standards for calibrating coaxial line measuring instruments such as six-port reflectometers and automatic network analysers. A need has developed for equivalent circuit models of offset O/Cs to assess their physical realisation.

This paper presents equivalent circuits for a range of air dielectric 14mm 50Ω coaxial line offset O/Cs which were produced using a computer program developed by the Engineering Department at Leicester University, [1] and [2].

2. OFFSET O/C STANDARDS

2.1 Fabrication

Coaxial line offset O/Cs are useful as known items for reflectometer and network analyser calibrations. However, a problem arises when attempting to realise calculable O/Cs in coaxial lines caused by the need to produce a well-defined abrupt transition from coaxial waveguide to circular waveguide.

This problem can be overcome in the 14mm 50Ω coaxial line size by removing the inner conductor spring contact (which screws into the threaded inner conductor) from the measuring instrument test port and screwing into it a suitably threaded length of precision inner conductor of the correct diameter and of the desired length. The outer shield of the coaxial line offset is produced by connecting a length of precision outer conductor of the correct diameter extending sufficiently beyond the point where the inner conductor is truncated so as to form an effectively infinite length of circular waveguide below cut-off. This produces a well defined O/C.

2.2 Modelling data

The Leicester University equivalent circuit computer program used complex voltage reflection coefficients (VRCs) produced by a sub-program from the VHF six-port software at NPL (Malvern). This sub-program gives VRCs for the offset O/C based on a lossy line model for the offset line.

The author is employed by Siemens Plessey Assessment Services Ltd and works under contract to the RF and Microwave Standards Section of DES, NPL, c/o DRA Electronics Division, RSRE, Malvern.
length, and a frequency dependent capacitor to represent the behavior of the O/C.

The lossy line model is based on a distributed circuit model comprising the four line elements R, L, G, and C; the series resistance and inductance, and shunt conductance and capacitance, respectively, all per unit length of transmission line, as shown in Figure 1:

![Equivalent circuit for a lossy coaxial line.](image)

Figure 1: Equivalent circuit for a lossy coaxial line.

For a lossless air-dielectric coaxial line, the resistance R, and conductance G, are zero, and the inductance L, and capacitance C, are independent of frequency. However, when the finite conductivity of the line is considered (i.e., a lossy line), the resistance is no longer zero, and along with the inductance, becomes frequency dependent. Expressions for the resistance and the inductance per unit length for the lossy line have been derived by Stratton [3] using the theory of electromagnetic fields and waves. These values, along with the usual expression for the line shunt capacitance are used in the six-port sub-program to model the offset line length.

Values for the discontinuity capacitance of a coaxial line O/C can be found in the literature eg: [4], [5] and [6]. The six-port sub-program uses values obtained from [6] to model the O/C. These values, along with the lossy line data, were used to produce VRCs for the Leicester University equivalent circuit program.

2.3 Choice of offset lengths

Several offset O/Cs of different lengths have been produced using the technique described in section 2.1, for use as primary impedance standards in the 14mm 50Ω coaxial line size. VRCs were generated for offset lengths corresponding to four of these offset O/Cs currently in use. Their measured lengths are: 4.165, 9.981, 14.965 and 20.001mm. VRCs were generated at frequencies of 10 to 300 MHz in 10 MHz steps (30 frequencies), corresponding to the operating frequency range of the VHF six-port reflectometer at NPL (Malvern).
3. EQUIVALENT CIRCUITS

The equivalent circuit produced by the Leicester University computer program for an offset O/C is given in Figure 2:

![Equivalent circuit diagram](image)

Figure 2: Equivalent circuit generated for an offset O/C.

The following Table gives the values for the equivalent circuit elements for the four different line lengths used:

<table>
<thead>
<tr>
<th>Offset line length (mm)</th>
<th>4.165</th>
<th>9.981</th>
<th>14.965</th>
<th>20.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_0 = 50 \Omega )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_1 ) (m( \Omega ))</td>
<td>4.105</td>
<td>24.536</td>
<td>60.371</td>
<td>131.18</td>
</tr>
<tr>
<td>( R_2 ) (k( \Omega ))</td>
<td>10.793</td>
<td>10.122</td>
<td>10.457</td>
<td>14.142</td>
</tr>
<tr>
<td>( L ) (pH)</td>
<td>37.592</td>
<td>11.256</td>
<td>6.442</td>
<td>4.643</td>
</tr>
<tr>
<td>( C ) (fF)</td>
<td>162.823</td>
<td>162.924</td>
<td>163.011</td>
<td>163.099</td>
</tr>
</tbody>
</table>

4. DISCUSSION

The symbol LN in Figure 2 represents a section of lossless 50\( \Omega \) coaxial line. Remembering that the data (VRCs) used by the computer program was generated using a lossy line model, the additional resistances and inductance present in the equivalent circuit might be interpreted as lossy elements added to the lossless line model in an attempt to model the lossy line data.

Since the offset coaxial lines have air as the dielectric, no deviation from the value for the lossless line is expected for any line shunt elements ie: the line will have a frequency independent shunt capacitance and zero shunt conductance. Therefore, it may be assumed that the capacitor in the equivalent circuit represents the discontinuity capacitance of the O/C, allowing this value to be compared with values found in the literature.
From [6], the discontinuity capacitance for this type of O/C is found to be 162.75fF at dc. At 300MHz, the value has increased by approximately 30aF, which is comparable with the estimated accuracy of the calculation. Therefore, the value of 162.75fF can be used for the O/C capacitance, and any frequency dependence ignored (at these frequencies).

Closer examination of the capacitance values in the equivalent circuit diagrams reveal that the capacitance is a function of the offset line length. This can be illustrated by plotting capacitance verses line length on a graph with linear axes as shown in Figure 3:

![Graph showing capacitance (from the equivalent circuit software) verses line length.](image)

The point of intersect on the vertical axis for this line is 162.75fF, which agrees with the value obtained from [6] and which was used to generate the VRCs in the six-port software.

To summarise, adding a coaxial line offset to the O/C introduces a discrepancy of 17.4aF/mm of offset line, which is thought to be due to subtle differences in the modelling of the offset line's shunt elements.

5. CONCLUSIONS

It has been shown that the Leicester University equivalent circuit computer program successfully modelled the coaxial O/C from VRCs supplied by the six-port calibration software. The largest deviation from the value in [6] was 350aF.
6. FUTURE INVESTIGATIONS

The next step in this research programme will be to provide the equivalent circuit computer program with measured values of VRC (with uncertainties) for the offset O/Cs examined in this document to see if the resulting equivalent circuit changes, and to see what range of values are produced for the circuit elements. This could be used as a guide to judge how well offset O/Cs can be realised in practice as well as to assess their suitability as primary impedance standards. These measurements will be made by the VHF six-port at NPL (Malvern).

7. ACKNOWLEDGEMENTS

The author would like to thank Mr A J Baden Fuller and Mr M Runham of Leicester University's Engineering Department for operating the equivalent circuit computer program and for useful discussions concerning all the work presented in this document. The author would also like to thank his colleague Mr J C Medley for providing the VRCs used by the equivalent circuit computer program.

8. REFERENCES


