Computer generated equivalent circuit models for coaxial-line offset open circuits

N.M. Ridler
J.C. Medley
A.J. Baden Fuller
M. Runham

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Abstract: Offset open circuits have been investigated at the National Physical Laboratory, NPL (Malvern), as primary impedance standards for calibrating coaxial line measuring instruments such as six-port reflectometers and automatic network analysers. A need has developed for equivalent-circuit models of offset open circuits to assess their physical realisation. The authors have analysed a range of air-dielectric 14 mm, 50 Ω coaxial-line offset open circuits and have produced an equivalent circuit whose elements are frequency independent and which separates the capacitance due to the open circuit from the effects of the length of the line.

1 Introduction

Coaxial-line offset open circuits are useful as known items for reflectometer and network-analysers calibrations. At high frequencies, the abrupt truncation of a coaxial line radiates and is not an electrical open circuit. However, a good open circuit can be constructed by the truncation of the inner conductor of the coaxial line with the outer conductor extending to form a section of empty circular waveguide. The circular waveguide is below cutoff and the junction between the coaxial line and circular waveguide produces a well defined open circuit.

Computer programs at the National Physical Laboratory, NPL (Malvern), provide a theoretical analysis of such an open circuit, generating an equivalent circuit whose elements are frequency dependent. A circuit generation program has been used to provide an equivalent circuit whose elements are frequency independent and which separates the capacitance due to the open circuit from the effects of the length of the line.

2 Offset open circuit standards

2.1 Fabrication

A problem arises when an attempt is made to realise physically calculable open circuits in coaxial lines, because of the need to produce a well defined abrupt transition from coaxial waveguide to circular waveguide. This problem can be overcome in the 14 mm, 50 Ω coaxial line by removing the inner conductor spring contact (which screws into the threaded inner conductor) from the measuring instrument test port and screwing into it a suitably threaded length of precision inner conductor of the correct diameter and of the desired length. The outer shield of the coaxial line is produced by connecting a length of precision outer conductor of the correct diameter extending sufficiently beyond the point where the inner conductor is truncated so as to form an effectively infinite length of circular waveguide below cutoff. This produces a well defined open circuit.

2.2 Modelling data

Complex voltage reflection coefficients (VRCs) were produced by a subprogram from the VHF six-port software at NPL (Malvern). This subprogram gives VRCs for the offset open circuit based on a lossy line model for the offset line length and a frequency dependent capacitor to represent the behaviour of the open circuit.

The lossy line model is based on a distributed circuit model comprising the four line elements \( R, L, G \) and \( C \); i.e. the series resistance and inductance, and shunt conductance and capacitance, respectively, all per unit length of transmission line, as shown in Fig. 1. For a lossless air-dielectric coaxial line, the resistance \( R \) and the conductance \( G \) are zero, and the inductance \( L \) and the capacitance \( C \) are independent of frequency.

Fig. 1 Equivalent circuit for lossy coaxial line

Expressions for \( R, L, G \) and \( C \) for a lossy line have been derived by Stratton [1] using the theory of electromagnetic fields and waves. In these expressions, \( R \) is nonzero and both \( R \) and \( L \) are frequency dependent. \( G \) and \( C \) are as in the lossless case. These expressions were

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Nick M. Ridler is with Siemens Plessey Assessment Services Ltd., and works under contract for the RF and Microwave Guided-Wave Standards Branch, Division of Electrical Science, National Physical Laboratory, RSRE, St. Andrew's Road, Malvern, Worcs. WR14 3PS, United Kingdom
John C. Medley is with the RF and Microwave Guided-Wave Standards Branch, Division of Electrical Science, National Physical Laboratory, RSRE, St. Andrew's Road, Malvern, Worcs. WR14 3PS, United Kingdom
A. John Baden Fuller and Michael Runham are with the Engineering Department, University of Leicester, University Road, Leicester LE1 7RH, United Kingdom

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used in the six-port subprogram to model the offset line length.

A recent paper by Daywitt [2] of the National Institute of Standards and Technology (NIST), Boulder, Colorado, presents new expressions for \( R, L, G \) and \( C \). Although the derivation of these new expressions is similar to that of Stratton's equations, they contain an extra frequency dependent correction term for each of the line elements. However, these correction terms are minimal for the frequencies of concern in this paper.

Values for the discontinuity capacitance of a coaxial line open circuit can be found in the literature [3–5]. The six-port subprogram uses values obtained from Reference 5 to model the open circuit. These values, along with the lossy line data, were used to produce VRCs at frequencies of 10–300 MHz in 10 MHz steps (30 frequencies), corresponding to the operating frequency range of the VHF six-port reflectometer at NPL (Malvern).

2.3 Choice of offset lengths

Several offset open circuits of different lengths have been produced using the technique described in Section 2.1, for use as primary impedance standards in the 14 mm, 50 \( \Omega \) coaxial line size. VRCs were generated for offset lengths corresponding to four of these offset open circuits currently in use. Their measured lengths are 4.165, 9.981, 14.965 and 20.001 mm.

3 Equivalent-circuit generation

3.1 The Program

A technique for the optimisation of electrical circuits has been developed at the University of Leicester [6] whereby a computer program removes components or adds new components and nodes to a circuit in an apparently random manner in order to achieve specified performance targets. The program minimises the error between the calculated performance of the circuit and a target performance. It has been used for the generation of equivalent circuits of microwave integrated circuit active and passive components [7] and for the design of microwave integrated-circuit transistor amplifiers [8].

Most measurements at microwave frequencies produce the performance as a set of s-parameters. For a two-port network, \( s11 \) is the input reflection coefficient, \( s21 \) is the forward transmission coefficient, \( s12 \) is the reverse transmission coefficient and \( s22 \) is the output reflection coefficient. For this investigation, the program target consisted of just the input reflection coefficient (\( s11 \)) in amplitude and phase.

Initially, the program is supplied with both the target performance data and a starting circuit. The initial circuit is first optimised with no topology change and is then progressively modified by the program using a succession of component and node additions and removals to produce a series of circuits each more nearly matching the target. The optimisation minimises the error between the calculated s-parameters of the circuit and the target s-parameters. The values of the components in the circuit are optimised using a weighted sum-of-squares error function derived from the individual s-parameter errors at each specified frequency. There are four distinct stages in the circuit optimisation:

Stage 3: Component addition: The program tests the effect of adding a new component between each pair of nodes in turn.

Stage 4: Node addition: The program tests the effect of adding a new component in series with each other component in turn or at the external nodes.

The program enters the component addition and the node addition routines alternately until a circuit of the required accuracy or complexity has been obtained.

3.2 Starting data

The program was presented with calculated reflection coefficient data at 30 frequency steps in the range 10–300 MHz. A simple starting circuit was used consisting of a length of ideal transmission line terminated in an ideal capacitor. The optimisation in the program had difficulties because it was possible to compensate for a variation in the length of the line by altering the value of the capacitance and vice versa. However, when the line length was fixed at its true value, the program was able to generate a satisfactory equivalent circuit.

The frequency independent equivalent circuit produced matched the target reflection coefficient very precisely (the error in phase < 10⁻⁵ degrees in each case).

3.3 Equivalent circuits

The equivalent circuit generated is shown in Fig. 2. The symbol LN in Fig. 2 represents a section of lossless 50 \( \Omega \) coaxial line. The values of the components in the circuit are given in Table 1.

The equivalent circuit matched the presented performance of each open circuit over the full range of frequencies 10–300 MHz. The line lengths were fixed to be

![Fig. 2 Equivalent circuit generated for offset open circuit](image)

<table>
<thead>
<tr>
<th>offset line length, mm</th>
<th>4.165</th>
<th>9.981</th>
<th>14.967</th>
<th>20.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 ), m( \Omega )</td>
<td>4.106</td>
<td>24.536</td>
<td>60.371</td>
<td>131.180</td>
</tr>
<tr>
<td>( R_2 ), m( \Omega )</td>
<td>10.793</td>
<td>10.122</td>
<td>10.457</td>
<td>14.142</td>
</tr>
<tr>
<td>( L ), ( \mu )H</td>
<td>37.892</td>
<td>11.256</td>
<td>6.442</td>
<td>4.643</td>
</tr>
<tr>
<td>( C ), fF</td>
<td>162.823</td>
<td>162.924</td>
<td>163.011</td>
<td>163.099</td>
</tr>
</tbody>
</table>

the actual lengths of the physical lines modelled. The values of the other components were determined by the program and gave the satisfactory results of an approximately constant value of the equivalent terminating capacitance.

4 Discussion

Remembering that the data (VRCs) used by the computer program were generated using a lossy line model, the additional resistances and inductance present in the equivalent circuit might be interpreted as lossy elements added to the lossless line model in an attempt to model...
the lossy line data. Since the offset open circuits have air as the dielectric, no deviations from the values for the lossless line are expected for the line shunt elements, i.e. the shunt capacitance should be independent of frequency and the shunt conductance should be zero.

From Reference 5, the discontinuity capacitance for this type of open circuit is found to be 162.75 fF at DC. At 300 MHz, the value has increased by approximately 30 aF, which is comparable with the estimated accuracy of the calculation. Therefore the value of 162.75 fF can be used for the open-circuit capacitance, and any frequency dependence can be ignored (at these frequencies).

Closer examination of the capacitance values in the equivalent circuit reveal that the capacitance is a function of the offset line length. This can be illustrated by plotting the capacitance against the line length on a graph with linear axes, as shown in Fig. 3. The point of intersec-

![Graph showing capacitance (from equivalent-circuit generation software) against line length](image)

FIG. 3 Graph showing capacitance (from equivalent-circuit generation software) against line length

...tion on the vertical axis for this line is 162.74 fF, which agrees with the value obtained from Reference 5 and which was used to generate the VRCs in the six-port software. Because the program has found it necessary to model the line with an inductance, some shunt capacitance is also necessary and may explain the variation in C shown in Fig. 3. Therefore it may be assumed that the capacitor in the equivalent circuit represents the discontinuity capacitance of the open circuit, allowing this value to be compared with those found in the literature.

To summarise, adding a coaxial line offset to the open circuit introduces a discrepancy of 17.4 aF/mm of offset line, which is thought to be due to subtle differences in the modelling of the offset line's shunt elements.

5 Conclusions

It has been shown that the Leicester University equivalent-circuit computer program successfully modelled the coaxial open circuit from VRCs supplied by the six-port calibration software. The largest deviation in discontinuity capacitance for the 20.001 mm offset open circuit from the value in Reference 5 was 350 aF.

6 Further investigations

The next step in this research program will be to provide the equivalent-circuit computer program with measured values of VRC (with uncertainties) for the offset open circuits examined in this document to see if the resulting equivalent circuit changes, and to see what range of values are produced for the circuit elements. This could be used as a guide to judge how well offset open circuits can be realised in practice as well as to assess their suitability as primary impedance standards. These measurements will be made by the VHF six-port at NPL (Malvern), this instrument having been recently introduced as the national standard impedance measuring system at VHF.

7 References