A Comparison of Microwave Reflection and Transmission Measurements in Rectangular Waveguide

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ABSTRACT
This paper reports on an exercise to compare measurements of reflection and transmission parameters in rectangular waveguide at microwave frequencies. The results obtained by the eight laboratories participating in the exercise are summarised in terms of the between-laboratory reproducibility. These values are compared with a within-laboratory repeatability assessment made by the pilot laboratory for the exercise. Multivariate robust statistical techniques have been used to obtain the summary statistics for the complex data.

1. INTRODUCTION
At the present time, the preferred measuring instrument for characterising microwave circuit parameters is the automatic network analyser (ANA). This multi-function measuring instrument finds application in almost every microwave measurement situation. The operating principles of these instruments are highly complex and an in-depth knowledge of these principles often eludes the practitioner. This brings into question an operator's reliance on these instruments to produce measurements of the required calibre.

In 1993, NPL (UK) set up ANAMET, the ANA METrology club for people and organisations interested in RF and microwave network measurements. One of the principle activities of the club is to organise measurement comparisons, to increase participants' confidence in making ANA measurements. The details of the comparisons are decided by the club membership and reflect areas in which they have most interest.

This paper reports on a comparison of reflection and transmission measurements in hollow metallic waveguide. This medium offers the most efficient, readily-available, means of transferring electrical energy at microwave frequencies. Of the various shapes and sizes currently in use, rectangular waveguide with approximate internal dimensions 23 mm × 10 mm is by far the most common. It is used in both military and commercial applications and propagates frequencies in the range 8.2 GHz to 12.4 GHz.

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1 Mr Ridler is employed by Assessment Services Ltd and works under contract to NPL (UK).
2. COMPARISON DETAILS
Three items were chosen for the comparison; a matched load, a mismatched load, and a 20 dB 3-port directional coupler. The participants were invited to supply measurements of the following five complex parameters:

(i) reflection coefficient of the matched load, nominally 0.0;
(ii) reflection coefficient of the mismatched load, nominally 0.2;
(iii) transmission coefficient of the through path of the coupler, nominally 0 dB;
(iv) transmission coefficient of the coupled path of the coupler, nominally 20 dB;
(v) transmission coefficient of the isolation path of the coupler, nominally 70 dB.

For each of the transmission measurements the remaining port was terminated with the matched load.

3. STATISTICAL ANALYSIS TECHNIQUES
Results were analysed, in accordance with international recommendations [1], in terms of the between-laboratory reproducibility of values and compared with a within-laboratory repeatability assessment made by the pilot laboratory, NPL Malvern. The reproducibility values indicate the level of variation found in the results supplied by the participating laboratories whereas the repeatability values indicate a typical variation made by one participant performing repeat measurements under essentially the same conditions. Therefore, the repeatability values provide a useful base-line to assess the variability in the participants’ measurements.

3.1 Robust estimators
It is conventional, when summarising data in a measurement comparison exercise, to give an average value and a measure of the dispersion of the data about that average. Usually the arithmetic mean and the standard deviation are used, however they provide a less useful summary for data sets containing unusual, or outlying, values — as was the case with this exercise. Under these circumstances, estimators exhibiting resilience (or, robustness) to outliers are preferable. For example, the median provides a robust average and the median absolute deviation (or its close relative, the inter-quartile range) provides a robust dispersion indicator. (These estimators were used to summarise data obtained in an earlier comparison exercise [2].)

The median, \( x_{\text{med}} \), for a set of results, \( x_i \), supplied by \( n \) participants, is simply the middle value of the results after they have been arranged in ascending order of size, \( i.e., x_1 \leq x_2 \ldots \leq x_n \). If the number of results is an even number, a unique middle value does not exist, so the median is the midpoint of the middle pair of values.

The median absolute deviation (MAD) is defined as follows;

\[
\text{MAD} = \text{median} \{ |x_i - x_{\text{med}}| \ ; i = 1, \ldots, n \}
\]

3.2 Bivariate considerations
Since all measured values in the exercise were complex (vector) quantities, further consideration was given to the statistical analysis of this type of data, i.e., bivariate
data. In particular, a bivariate version of the median — the spatial median — was used to establish an average value for each data set.

The spatial median is defined as the point in the complex plane which minimises the sum of the absolute differences (distances) between the individual participants’ values and its value. Expressed mathematically, the spatial median, \( \mu \), minimises:

\[
\sum_{i=1}^{n} |X_i - \mu|
\]

where \( X_1, \ldots, X_n \) are the complex values supplied by the participants.

The spatial median provides an average value which is relatively unaffected by the presence of outliers in complex data. (This is analogous with the resilience exhibited by the conventional median for univariate data.)

3.3 MAD calculations

When assessing the variability in the complex reflection and transmission measurements, the participants are mainly concerned with the variation in the magnitude and phase components of the measurement parameters. The MAD calculation has therefore been applied separately to the magnitude and phase components of each data set (with respect to the magnitude or phase of the spatial median, respectively). I.e.

\[
\text{MAD (Magnitude)} = \text{median} \{ |X_i| - |\mu| ; i = 1, \ldots, n \}
\]

\[
\text{MAD (Phase)} = \text{median} \{ |\phi_{X_i} - \phi_\mu| ; i = 1, \ldots, n \}
\]

where \( \phi_{X_i} \) and \( \phi_\mu \) are the phase values for the \( i \)th result and the spatial median, respectively. (Arithmetic performed on phase data requires special consideration as values are usually represented on a scale which is periodic in nature, i.e., either 0° to 360°, or \( \pm 180° \). This requires a form of clock arithmetic.)

As a measure of dispersion, \( \pm \text{MAD (Magnitude)} \) and \( \pm \text{MAD (Phase)} \) produce intervals about the median value which are expected to contain half the results, in each case.

4. RESULTS SUMMARIES

The results summaries supplied to the participants in the exercise included the following information, at each frequency; (i) the value of the spatial median, (ii) the difference between the participants value and the spatial median value, and (iii) the MAD values. However, for the purposes of this paper, a more concise summary is adequate concentrating on the variations between the participants values (the reproducibility) and comparing with the variations recorded during the repeatability assessment made by the pilot laboratory.

Since no frequency dependence was discernible with the reported values, a single MAD value for each measurement configuration provides an adequate summary. The summary statistics for the reflection and transmission measurements are given below.
MAD Repeat summarises the repeatability variations and MAD Repro the reproducibility variations.

<table>
<thead>
<tr>
<th>Reflection coefficient</th>
<th>Linear magnitude</th>
<th>Phase (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAD Repeat</td>
<td>MAD Repro</td>
</tr>
<tr>
<td>Matched load</td>
<td>0.0005</td>
<td>0.0090</td>
</tr>
<tr>
<td>Mismatched load</td>
<td>0.0002</td>
<td>0.0156</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transmission coefficient</th>
<th>Log magnitude (dB)</th>
<th>Phase (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAD Repeat</td>
<td>MAD Repro</td>
</tr>
<tr>
<td>Through path</td>
<td>0.004</td>
<td>0.015</td>
</tr>
<tr>
<td>Coupled path</td>
<td>0.004</td>
<td>0.023</td>
</tr>
<tr>
<td>Isolation path</td>
<td>0.1</td>
<td>2.6</td>
</tr>
</tbody>
</table>

5. DISCUSSION

5.1 Sources of error
As a rule of thumb, repeatability figures indicate typical variations due to the effects of random errors in the measurements. Similarly, the reproducibility figures indicate typical variations due to the combined effects of random and systematic errors.

Since the reproducibility statistics are generally between two and eight times larger than the repeatability statistics this indicates that systematic errors are significant (and can dominate) for these types of measurements. (The summary statistics for the measurements of the transmission of the isolation path are discussed separately, below.)

5.2 Problems in analysing the isolation path values
The measurement data supplied by the participants for the isolation path of the coupler has caused some analysis problems as in each data set there appeared to be two distinct groups of data points. The groups have similar magnitudes, but have phase values nominally 180° apart. (See, for example, Figure 1.) Under these circumstances, the use of the MAD alone as an indication of variation in the data sets is inadequate. Further work is needed to provide an acceptable summary for data sets of this kind.

6. CONCLUSIONS
The comparison exercise produced some interesting results which have raised the awareness of the difficulties in making measurements of this kind. In particular, the indication of significant (and potentially dominant) systematic errors means that careful consideration should be given to the assessment of measurement uncertainty for such measurements.
The comparison has also demonstrated a problem with measuring the phase of waveguide transmission measurements using ANAs (the isolation path measurements). It has been shown elsewhere [3] that a 180° phase ambiguity can occur with this type of measurement.

Finally, the knowledge gained by the participants in this comparison exercise should provide valuable insight when quantifying uncertainty and assessing the ability of these instruments to perform measurements of this kind over this very important frequency range.

7. ACKNOWLEDGEMENT
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REFERENCES

Figure 1  The eight measurements of the transmission coefficient of the isolation path of the coupler at 11 GHz. The results are represented by black dots in the complex transmission coefficient plane.