EVALUATING THE UNCERTAINTY IN MEASUREMENTS - GENERAL PRINCIPLES

N M Ridler¹

Abstract

This paper presents general principles involved in evaluating, and expressing, the uncertainty of the result of a measurement. Each stage in the overall evaluation process is discussed and illustrated using a simple example. The approach given in the paper is in line with most of the current national and international recommendations.

Introduction

Measurement attempts to determine the true value of a characteristic for a device. Since all measurement processes are affected by errors, some degree of error will inevitably be present in the measurement result, making it an approximation of the true value. An important part of an overall measurement process is the identification of all the contributing errors and, where possible, making the necessary corrections. Where corrections are not possible, the effect the remaining errors have on the measurement result needs to be evaluated. This is the uncertainty evaluation process for the measurement. The resulting value for the measurement's uncertainty indicates quantitatively the doubt about the accuracy of the result. This allows the user of the result to assess its reliability and enables meaningful comparison with other results.

This paper discusses the general principles involved in evaluating and expressing uncertainty in measurement. Each stage of the process is discussed, including identifying, evaluating and combining the uncertainty contributions, and expressing the overall uncertainty. An example is given to illustrate each step of the process. This example concerns a measurement, made using an impedance bridge, of the resistance at 250 MHz of a nominal 100 Ω resistor. The example is kept simple for clarity.

Step 1 - Understand the measurement process and identify sources of error

One of the most important aspects of an uncertainty evaluation exercise is the need for a detailed understanding of the overall measurement process. Such understanding is required to ensure all of the potential contributions to the measurement's uncertainty are identified. This means the design engineer, or skilled operator, of the measurement system is often best suited to perform the evaluation exercise.

The exercise begins by examining in detail the measurement process and identifying the potential sources of error. This often involves representing the measurement process using a variety of means, including: flow diagrams; block diagrams; computer simulations; the use of a mathematical model; etc.

¹ Mr Ridler is employed by Assessment Services and works under contract to the National Physical Laboratory, and is based at DRA Malvern, Worcs, WR14 3PS.

© Crown copyright 1996. Reproduced by permission of the Controller of HMSO.
Identified errors should be corrected, where possible, e.g., by performing an instrument calibration. Where correction is either not possible or incomplete, the remaining error is treated as a contribution to the measurement's uncertainty.

Example:- In order to study the measurement process in detail, the bridge was represented using a block diagram. This led to the following four sources of error being identified:

(i) the connection repeatability of the resistor to the bridge;
(ii) the electrical noise present on the signals detected by the bridge;
(iii) the uncertainty accompanying the transfer standard used to calibrate the bridge;
(iv) the accuracy of the bridge's frequency source.

In this case, all four sources of errors could not be corrected (i.e., remain after calibration), so they were treated as contributions to the measurement's uncertainty.

Step 2 - Quantify each uncertainty contribution

Having identified all the contributions to the uncertainty in the measurement process, each contribution requires evaluation. This means estimating a figure to represent the likely variation in the value for the contribution. Nowadays, the recommended statistic for providing such a figure of variation is the standard deviation. A contribution evaluated as a standard deviation is called the standard uncertainty for the contribution.

Other measures of variation can usually be converted into an equivalent standard uncertainty using a suitable scaling factor. For example, variations in a parameter quoted as limits of variation can be converted to a standard uncertainty by dividing the limit value by $\sqrt{3}$. Similarly, variations quoted as intervals at a level of confidence of 95%\(^2\) (e.g., on calibration certificates) can usually be converted to a standard uncertainty by dividing the interval by 2.

Example:- The uncertainty associated with the transfer standard (a coaxial capacitor) used to calibrate the bridge is taken from its calibration certificate, where it is quoted as ±0.56 pF with a level of confidence of 95%. We divide this value by 2 to give an equivalent standard uncertainty of ±0.28 pF.

The frequency source accuracy was quoted in the manufacturer's specification for the source as ±10 Hz. This is assumed to represent limits for this quantity, so we divide this limit by $\sqrt{3}$ to give an equivalent standard uncertainty of approximately ±6 Hz.

Step 3 - Evaluate the effect of the uncertainty contribution on the final result

The next step is to determine the effect each contribution has on the final measurement result. This can be a difficult process involving a variety of methods, often including experimentation and/or experience. (If the measurement process has been represented using a mathematical model, then partial differentiation can be used to perform this step of the process.)

\[^2\] An interval quoted at a level of confidence of 95% implies that the uncertainty interval has a 95% chance of containing the true value for the measurement. In other words, in 19 times out of 20 the uncertainty interval should contain the true value.
Example:- The contribution from the transfer standard can be assessed by calibrating the bridge under two conditions; firstly using the value for the transfer standard given on its calibration certificate, then calibrating again using this value plus its standard uncertainty. The difference between the two subsequent resistance measurement results given by the bridge represents the standard uncertainty contribution, which was found to be \( \pm 0.26 \Omega \).

The same technique can be used for the contribution from the accuracy of the frequency source. Its standard uncertainty was found to be \( \pm 0.019 \Omega \).

(Note, these contributions are now in the same units as the final measurement result, i.e., ohms.)

Sometimes steps two and three, above, can be combined, i.e., variations in the contribution can be monitored simultaneously with the effects the variations have on the measurement result.

Example:- This can be done for the contribution due to electrical noise by measuring the item repeatedly under essentially the same conditions. The standard deviation of the mean of the repeated measurement results can be used as the standard uncertainty for this component.

Alternatively, the electrical noise and connection repeatability contributions can be evaluated simultaneously using a similar method, but also disconnecting and re-connecting the resistor between repeat measurements.

The contribution due to connector repeatability and electrical noise, using this technique, was calculated as a standard uncertainty and was found to be \( \pm 0.097 \Omega \).

Step 4 - Combine the contributions to give the overall uncertainty

Having identified each contribution and its effect on the measurement result, we need to combine the contributions to give an overall figure for the measurement's uncertainty, called the expanded uncertainty. In most cases, this can be accomplished by:

(i) summing the squared standard deviations of all the contributions;
(ii) taking the square-root of this sum;
(iii) multiplying the square-root by a numeric factor called the coverage factor.

The coverage factor is either established by convention or chosen to give the required level of confidence for the result. For example, a coverage factor of two is typical, and will often correspond approximately to a level of confidence of 95%. If greater confidence is required, a larger coverage factor should be used.

Example:- For our example we have;

\[ \text{Expanded uncertainty} = k \left[ (0.26)^2 + (0.019)^2 + (0.097)^2 \right]^{\frac{1}{2}} \Omega. \]

where \( k \) is the coverage factor. If we use \( k = 2 \), then the expanded uncertainty for the resistance measurement is \( \pm 0.56 \Omega \).

Step 5 - Reporting the result

Having evaluated the uncertainty for the measurement, it is important to report the value in an unambiguous manner which can be easily understood, and used, by the customer. Current guidelines generally recommend quoting the expanded uncertainty alongside the result of the measurement, and to include the value of the coverage factor used, and the approximate level of confidence.
**Example:** Assuming the measured value of the resistor was found to be 100.24 Ω, our example could be reported as follows:

Resistance at 250 MHz = (100.24 ± 0.56) Ω.

the reported uncertainty is the expanded uncertainty using a coverage factor of $k = 2$, providing a level of confidence of approximately 95%.

**Discussion**

An uncertainty evaluation process can be extremely complicated or very simple. For measurement systems whose accuracy is governed predominantly by the transfer standard used for its calibration, this can be the only contribution requiring consideration. Our example illustrates this.

**Example:** If we re-calculate the expanded uncertainty omitting the uncertainty due to source frequency accuracy, the answer is still ±0.56 Ω. Clearly, this contribution can be ignored. Indeed, if we were to leave out both the frequency accuracy contribution and the electrical noise/connection repeatability contributions, the calculated uncertainty would only be reduced to ±0.52 Ω. This is equivalent to assuming all uncertainty in the measurement is due to the uncertainty accompanying the transfer standard used to calibrate the bridge.

The reduced uncertainty produced by omitting contributions can often be compensated for by reducing the number of significant figures used to express the overall uncertainty and rounding the number pessimistically, i.e., by rounding up. This would cause our value of ±0.52 Ω to be rounded up to ±0.6 Ω.

Successful identification of components whose contribution to the overall uncertainty is negligible can help reduce considerably the complexity of the uncertainty evaluation process. It is important however, to demonstrate that a given contribution has a negligible effect on a result before omitting it from the calculations.

**Summary**

This paper has presented some of the more general principles one might encounter during a typical uncertainty evaluation process. Most of the guidelines currently available conform, broadly speaking, to these general principles. Departures will inevitably occur, both in written standards and practical aspects of uncertainty evaluation, but these departures are a consequence of the wide variety of situations to which these principles are applied.

**Recommended further reading**


