Coverage factors and levels of confidence for vector ANA measurements

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Introduction

These days, documents giving guidance on expressing uncertainty in measurement, e.g. [1, 2], usually recommend evaluating the standard uncertainty in a measurement, and then, using a coverage factor, to quote an expanded uncertainty at a specific level of confidence. Typically, in RF and microwave measurement, it is recommended that a coverage factor of $k = 2$ is used, and, it is said, this produces a level of confidence of approximately 95%. This means that the expanded uncertainty describes an interval about the measurement result which, we can say with 95% confidence, contains the notional 'true value' for the thing being measured (i.e., the measurand).

This works well, in general, for scalar measurements (e.g., attenuation, return loss, VSWR, etc) where the concept of an interval of uncertainty can be easily applied and understood. However, with $s$-parameter measurements made on ANAs, the measurand is a vector, two-dimensional, quantity. Such values are usually represented as points in the complex plane and this makes the concept of an interval of uncertainty more difficult to apply. Instead, we need to think in terms of two-dimensions, and this leads to considering the uncertainty of measurement as a region of uncertainty which should contain the 'true value' at a specified level of confidence. Unfortunately, regions of uncertainty (e.g., for standard uncertainty, expanded uncertainty, etc) for vector measurements do not scale (i.e., multiply up) by the same amount as intervals of uncertainty for scalar measurements. This means that, for standard uncertainties and expanded uncertainties, the correspondence between $k = 2$ and a 95% level of confidence is no longer, in general, valid.

This Note illustrates the above problem by using a very simple shape - a circle - to represent the region of uncertainty for a vector ANA measurement. (Incidentally, such circles are currently used to represent the uncertainty in the $s$-parameter measurements made for customers of the primary RF and microwave impedance measurement facility at NPL.) Use is made in this Note of information given in an earlier ANAMET Report [3] to relate coverage factors and levels of confidence for uncertainty 'circles'.

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The circular normal distribution

For scalar measurements, uncertainty intervals are often related by the assumption that the normal, or Gaussian, distribution characterises the measurement result. The simplest extension of this view, for a vector measurement, is to consider a circular region of uncertainty related to a circular normal distribution for the measurement result. Such a distribution, when viewed from either the real or imaginary axis, will 'look like' a conventional normal distribution with the same shape and width. As such, the distribution possesses only a single value for the standard deviation (i.e., a value common to both real and imaginary components) and this is analogous with the radius of the circular region of uncertainty.

The expression relating coverage factor, $k$, to level of confidence, $p$, for a circular normal distribution, is as follows:

$$ p = 1 - e^{-k^2/2} $$

This expression can be used to compare coverage factors and levels of confidence for vector ANA measurements with equivalent values, for scalar measurements, found in the literature [1]. These values are given below.

<table>
<thead>
<tr>
<th>Coverage factor</th>
<th>Level of confidence</th>
<th>Level of confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scalar measurement</td>
<td>Vector ANA measurement</td>
</tr>
<tr>
<td>$k = 1$</td>
<td>68%</td>
<td>39%</td>
</tr>
<tr>
<td>$k = 2$</td>
<td>95%</td>
<td>87%</td>
</tr>
<tr>
<td>$k = 2.45$</td>
<td>99%</td>
<td>95%</td>
</tr>
</tbody>
</table>

Discussion

From the above table, it is clear that using a coverage factor of $k = 2$ for vector ANA measurements provides a level of confidence of 87%, and not 95%! To achieve a 95% level of confidence for such measurements, a coverage factor of $k = 2.45$ should be used. For practical purposes, perhaps this could be rounded pragmatically to $k = 2.5$.

However, the question as to whether we should use $k = 2$, giving a level of confidence of 87%, or, $k = 2.5$, giving a level of confidence of 95%, when expressing uncertainty in vector ANA measurements, remains open to debate.

References

