A GENERALISED TREATMENT OF THE CALIBRATION AND MEASUREMENT FOR VECTOR-INDICATING MICROWAVE REFLECTOMETERS

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Abstract
This paper gives a generalised treatment of the calibration and measurement of vector-indicating microwave reflectometers. The reflection coefficient is treated as a complex measurement with an elliptical region of uncertainty. The reflectometer calibration is treated as (i) a deterministic problem, and (ii) an over-determined problem employing Generalised Distance Regression.

Reflectometer Theory
The relationship between the “true” value of the reflection coefficient, $\Gamma$, of a device and the value indicated by the non-ideal reflectometer, $W$, used to measure it, is given by

$$\Gamma = \frac{aW + b}{cW + 1}, \quad (1)$$

where $a$, $b$, and $c$ are the calibration constants that describe the internal network of the non-ideal reflectometer. The values of $a$, $b$, and $c$ are determined by calibrating the system. This can be achieved by connecting three or more calibration standards whose reflection coefficients are assumed to be known. Thus, we have the following set of equations,

$$\Gamma_i = \frac{aW_i + b}{cW_i + 1}, \quad i = 1, \ldots, m, \quad (2)$$

where $m$ is the number of standards, the set of $\Gamma_i$ are the known reflection coefficients of the standards, and $W_i$ are the corresponding reflection coefficients indicated by the reflectometer. By solving this system of equations, the calibration constants can be determined and any subsequent measurement can be corrected. Note that all variables are complex numbers, and can be written in terms of their real and imaginary parts:

$$\Gamma = x + jy,$$
$$W = u + jv,$$
$$a = a_R + ja_Z,$$
$$b = b_R + jb_Z,$$
$$c = c_R + jc_Z. \quad (3)$$

Calibration using three standards
In this section we shall consider the case where $m = 3$. This is equivalent to setting up three equations of the form (2) for $i = 1, 2, 3$. This number of equations is the minimum sufficient to determine the three complex-valued calibration constants $a$, $b$, and $c$. Substituting the expressions (3) into the equations (2), and equating real and imaginary parts, leads to a linear system of equations for the required calibration constants of the form

$$H\gamma = t, \quad (4)$$

where

$$\gamma = (a_R, a_Z, b_R, b_Z, c_R, c_Z)^T,$$
$$t = (x_1, y_1, x_2, y_2, x_3, y_3)^T,$$

and $H$ is a 6-by-6 matrix. This equation is solved by a stable numerical method of solution. Furthermore, we may obtain estimates for the uncertainty in the calibration constants by making use of uncertainty estimates in the calibration standards. If the covariance matrix for the real and imaginary parts of the calibration standards is written as $V(s)$, where

$$s = (u_1, v_1, x_1, y_1, u_2, v_2, x_2, y_2, u_3, v_3, x_3, y_3)^T,$$

then the covariance matrix for the calibration constants is given by

$$V(\gamma) = JV(s)J^T, \quad (5)$$

where $J$ is the Jacobian matrix of $\gamma$ with respect to $s$ [1].

Calibration using more than three standards
In this section we indicate how to model the calibration of the reflectometer using more than the minimum number of three standards. This results in an overdetermined system of equations to be solved, and can potentially lead to increased levels of accuracy in the calibration process. The mathematical details are provided in [1], and the problem is formally to minimise

$$e^TV^{-1}e \quad (6)$$
with respect to the parameters \( a_{\text{rs}}, a_{\text{ss}}, b_{\text{rs}}, b_{\text{ss}}, c_{\text{rs}}, c_{\text{ss}} \)
and \( u_i^*, v_i^*, x_i^*, y_i^*, i = 1, \ldots, m \), subject to the equality constraints

\[
\Gamma_i^* = \frac{a W_i^* + b}{c W_i^* + 1}, \quad i = 1, \ldots, m,
\]

where

\[
e = (u_1^* - u_1, v_1^* - v_1, x_1 - x_1^*, y_1 - y_1^*, \ldots, u_m^* - u_m, v_m^* - v_m, x_m - x_m^*, y_m - y_m^*)^T,
\]

and \( V \) is the covariance matrix for the vector \( e \). The formulation of the problem in this form is an example of Generalised Distance Regression (GDR), algorithms for the solution of which can be found in [2, 3]. Solving the problem leads to values for the calibration constants and estimates for their associated uncertainties, as in the calibration process using three standards.

**Obtaining measurements from observed values**

The above considerations permit, in principle, the reflection coefficient of an unknown DUT for both the determined \((m = 3)\) and overdetermined \((m > 3)\) cases to be obtained. Assuming an observed value for the reflection coefficient of a DUT, \( W_0 \), we use this, together with the determined calibration constants in the original calibration equation, (1), to evaluate the "true" calibration coefficient,

\[
\Gamma_0 = \frac{a W_0 + b}{c W_0 + 1}.
\]  

An estimate for the covariance matrix for the uncertainties in the output can also be obtained, [1],

\[
V(\Gamma_0) = CV(q)C^T,
\]

where

\[
q = (W_{\text{rs}}, W_{\text{ss}}, y_i^T)^T,
\]

and \( C \) is the Jacobian matrix for the system, whose explicit form can be seen in [1].

**Results and Conclusions**

Results have been obtained from a number of simulated and experimental examples using this technique, and are presented in [1]. We have performed calibration using three, four and five standards, where in the latter case the calibration coefficients are given by \( \Gamma = (-1, 0, +1, -j, +j) \), and have seen corresponding reductions in uncertainty estimates for obtained measurements. It is expected that additional standards would further improve the measurement accuracy.

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**References**

