COMPARISON OF COMPLEX SCATTERING COEFFICIENT MEASUREMENTS OF A MICROWAVE STEP ATTENUATOR

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Abstract

This paper presents results obtained from a recent measurement comparison exercise of complex scattering coefficients of a microwave step attenuator. The results obtained by the eight participating laboratories are summarised in terms of the between-laboratory reproducibility. Multivariate robust statistical techniques are used to summarise the complex-valued data.

Comparison details

Eight measurement laboratories chose to participate in the comparison exercise, including seven national metrology institutes. The comparison comprised a coaxial step attenuator fitted with female precision 3.5 mm connectors (i.e. a non-insertable device). The step attenuator was measured as a two-port device ($S_{21}$) at 0 dB, 10 dB and 70 dB settings, and as a one-port device ($S_{11}$) at the same settings, terminated with a short-circuit. This produced six different configurations which were measured at frequencies of 1 GHz to 26.5 GHz in 1.5 GHz steps. The results were expressed in terms of the linear magnitude and phase of each complex scattering coefficient.

The comparison exercise was coordinated by ANAMET - the Automatic Network Analyser METrology forum for people and organisations involved in RF and microwave network measurements.

Statistical analysis techniques

Results have been analysed, in accordance with international recommendations [1], in terms of the between-laboratory reproducibility of values. The reproducibility values indicate the level of variation found in the results supplied by the participating laboratories.

Robust estimators

It is conventional, when summarising data in a measurement comparison exercise, to give an average value and a measure of the dispersion of the data about that average. Usually the arithmetic mean and the standard deviation are used, however they provide a less useful summary for data sets containing unusual, or outlying, values - as was the case with this exercise. Under these circumstances, estimators exhibiting resilience (or, robustness) to outliers are preferable. For example, the median provides a robust average and the median absolute deviation provides a robust dispersion indicator (used here to indicate the between-laboratory reproducibility).

The median, $\bar{x}_{med}$ for a set of results, $x_i$, supplied by $n$ participants, is simply the middle value of the results after they have been arranged in ascending order of size, i.e. $x_1 \leq x_2 \ldots \leq x_n$. If the number of results is an even number, a unique middle value does not exist, so the median is taken as the midpoint of the middle pair of values.

The median absolute deviation (MAD) is defined as follows:

$$MAD = median\{|x_i - \bar{x}_{med}| ; i = 1, \ldots, n\}$$

Bivariate considerations

Since all measured values in the exercise were complex (vector) quantities, further consideration was given to the statistical analysis of this type of data, i.e. bivariate data. In particular, a bivariate version of the median - the spatial median - was used to establish an average value for each data set.

The spatial median is defined as the point in the complex plane which minimises the sum of the absolute differences (distances) between the individual participants' values and its value. Expressed mathematically, the spatial median, $\mu$, minimises

$$\sum_{i=0}^{n} |X_i - \mu|$$

where $X_0, \ldots, X_n$ are the complex values supplied by the participants.

The spatial median provides an average value which is relatively unaffected by the presence of outliers in complex data. This is analogous with the resilience exhibited by the conventional median for univariate data.

MAD calculations

When assessing the variability in the complex scattering coefficient measurements, the participants are mainly concerned with the variation in the magnitude and phase components of the measurement parameters. The MAD calculation has therefore been applied separately to the magnitude and phase components of each data set (with respect to the magnitude or phase of the spatial median, respectively), i.e.

$$MAD (\text{Magnitude}) = median\{||X_i| - |\mu|| ; i = 1, \ldots, n\}$$

$$MAD (\text{Phase}) = median\{\phi_{X_i} - \phi_{\mu} ; i = 1, \ldots, n\}$$

where $\phi_{X_i}$ and $\phi_{\mu}$ are the phase values for the $i^{th}$ result.
and the spatial median, respectively. (Note that arithmetic performed on phase data requires special consideration as values are usually represented on a scale which is periodic in nature, i.e. either 0° to 360°, or ±180°. This requires a form of modular, or clock, arithmetic.)

As a measure of dispersion, ± MAD (Magnitude) and ± MAD (Phase) produce intervals about the median value which are expected to contain half the results, in each case.

**Results summaries**

The results summaries supplied to the participants in the exercise included the following information, at each frequency: (i) the value of the spatial median, (ii) the difference between the participants' value and the spatial median value, and (iii) the MAD values. However, for the purposes of this paper, a more concise summary is appropriate concentrating on the variation found in the magnitude MAD values. (Note that variation in the phase values will relates directly to the magnitude of the scattering coefficient vector.)

The maximum MAD values obtained for the six measurement configurations are given in Tables 1 and 2. These tables also show the range of measurement values produced by each configuration at all frequencies of the comparison.

<table>
<thead>
<tr>
<th>Attenuator setting (dB)</th>
<th>Range of measurement values</th>
<th>Maximum MAD value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.66 to 0.95</td>
<td>0.0050</td>
</tr>
<tr>
<td>10</td>
<td>0.031 to 0.17</td>
<td>0.0025</td>
</tr>
<tr>
<td>70</td>
<td>0.0067 to 0.10</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

Table 1: $S_{21}$ values.

<table>
<thead>
<tr>
<th>Attenuator setting (dB)</th>
<th>Range of measurement values</th>
<th>Maximum MAD value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.80 to 0.98</td>
<td>0.0021</td>
</tr>
<tr>
<td>10</td>
<td>0.25 to 0.31</td>
<td>0.0007</td>
</tr>
<tr>
<td>70</td>
<td>0.00025 to 0.00031</td>
<td>0.0000084</td>
</tr>
</tbody>
</table>

Table 2: $S_{21}$ values.

**Observations**

The maximum MAD values shown above compare favourably with equivalent values obtained during an earlier comparison exercise [2]. This might suggest that the data obtained during this exercise was generally well-behaved.

However, by examining plots of the data obtained during this comparison exercise, it was found that there were occasional systematic differences between some participant's values.

This can be seen in Figure 1, which shows the difference in the $S_{21}$ phase values from the spatial median. It can be seen that one participant's set of values shows a frequency dependent difference from the majority of values scattered about the spatial median.

![Figure 1: $S_{21}$ phase value differences, showing one participant's values diverging from the majority of values, as a function of frequency.](image)

**Conclusions**

This comparison exercise has produced some useful summary information concerning the likely variation to be found in measurements of this kind. In addition, the presence of occasional unusual values has demonstrated the advantage in using robust statistical procedures for analysing such data. For the complex-valued data examined here, these robust methods have been extended to deal with multivariate measurement data.

Finally, the knowledge gained by the participants in this comparison exercise should provide valuable insight when assessing the uncertainty in the measurements.

**Acknowledgement**

The work presented in this paper was funded by the National Measurement System Policy Unit of the UK Government's Department of Trade and Industry.

**References**
