Abstract - In situ resistivity measurements on beadless coaxial air lines allow the determination of the wavelength within the air line using only information on the diameters of the conductors. From the measured phase shift of such an air line the calculated electrical length of the air line shows very good agreement with geometrical values. This indicates that these lines could be used as verification devices for electrical phase measurements (e.g. of microwave S-parameters).

I. Introduction
In the past, air lines were proposed mainly as standards for characteristic impedance (see e.g. [1]). As such, their use is promoted in international guidelines on assessing Vector Network Analysers (VNA) [2]. It is known that at low frequencies deviations from idealized behaviour occur and that corrections should be made. During the writing of [2] the question of uncertainty in phase measurements was considered. However, most contributors felt that a simple conversion from amplitude uncertainty into phase uncertainty was insufficient: measurements show that at least the statistical distribution is sometimes dominant in amplitude, sometimes in phase. Realising that an air line has a well defined mechanical length the possibility of using air lines as phase standards should be investigated. This paper describes investigations done at two laboratories on the suitability of a simple method of implementation.

II. Phase changes as function of frequency
Air lines are usually described in terms of their high frequency (lossless) limit with the following relations for the characteristic impedance, $Z_0$, and the propagation constant, $\gamma_0$,

$$Z_0 = 59.94 \ln \left( \frac{a}{b} \right)$$

and

$$\gamma_0 = \frac{j \cdot 2\pi}{\lambda}$$

where $\lambda$ is the wavelength, and $a$ and $b$ are the radii of the center and outer conductors, respectively. This shows that the change in phase is linear with frequency and with the length of the air line. Figure 1 shows the result of a measurement of the phase on an air line: a linear phase correction (often termed electrical delay) is applied to obtain a more or less constant value at high frequencies.

The electrical length of the air line determined from the phase measurement differs by more than 40 $\mu$m from its calibrated geometrical length. This aspect combined with the curved shape of the data presented in figure 1 requires a more detailed investigation before traceability to geometrical values is possible.

III. Model for a lossy air line
In the real world, resistivity of the conductor material should be taken into account. Calculations using the formulas given in [3] already show that significant changes in phase may occur for realistic values of resistivity (e.g. $8 \times 10^{-8}$ $\Omega$m for BeCu), see figure 2.

A search of the literature shows that sometimes a wide range of values are given, even for pure bulk material. Hence it is more realistic to look for an experimental method to obtain the value for the air line used.

IV. Resistivity from loss measurements
In air lines the reflection is very low, which means that the transmission is determined completely by absorption in the air line – i.e. the loss due to the skin depth. This means that an overall resistivity, $\rho$, can be determined (at each frequency, $f$) from S-parameter measurements using a VNA. The following equations are used [4,5] for this calculation:
\[
\alpha = -\ln\left|S_{21}\right|/l \tag{3}
\]
\[
\rho = \frac{200 \alpha \cdot b}{1 + b/a} \cdot \frac{\mu}{\mu + \mu_0} \cdot \frac{\pi}{f} \tag{4}
\]

where \(l\) is the physical length of the line, \(\alpha\) is the attenuation constant and \(\mu\) is the permeability. In figure 3 an example is given for an air line from a VNA verification kit. It shows that the value is more or less independent of frequency, so in our modeling we can use a single value for determining the propagation constant and the characteristic impedance of this air line.

![Figure 3. Measured resistivity of a 100 mm PC7 air line.](image)

From the transmission measurements the resistivity for each air line is obtained. In Table 1 typical values found for four commonly used line sizes are given. The air lines are from different manufacturers. Different calibration schemes (TRL and SOLT) were used: however, this had no significant effect on the determined values of resistivity. VSL found different values for lines from different manufacturers. Data from NPL are taken from [5].

![Table 1: Measured resistivities for lines in four different line sizes](image)

**Table 1: Measured resistivities for lines in four different line sizes**

<table>
<thead>
<tr>
<th>Connector type</th>
<th>Length (mm)</th>
<th>(\rho) (NPL) n(\Omega) m</th>
<th>(\rho) (VSL) n(\Omega) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC7</td>
<td>100</td>
<td>118</td>
<td>77-120</td>
</tr>
<tr>
<td>N50</td>
<td>125</td>
<td>124</td>
<td>120</td>
</tr>
<tr>
<td>PC3.5</td>
<td>75</td>
<td>117</td>
<td>60</td>
</tr>
<tr>
<td>K (2.92)</td>
<td>75</td>
<td>-</td>
<td>70</td>
</tr>
</tbody>
</table>

**V. Calculated electrical length**

For each individual air line the resistivity value, in combination with the measured center and outer conductor diameter values, is used to calculate the wavelength. For comparison purposes, the wavelength is also calculated for the lossless case (i.e. assuming \(\rho = 0\)). Next, for both cases, the electrical length of the air line is calculated using the measured phase shift. These results are compared with the geometrical length of the air line (after correcting for any temperature difference between the laboratories that performed the geometrical and electrical measurements).

![Figure 4. Comparison between electrical and geometrical determinations of length for a 100 mm PC7 air line.](image)

Figure 4 shows a comparison between geometrical and electrical lengths, in terms of the difference between the two electrical methods and the geometrical one. The electrical method using the resistivity data shows very good agreement with the measured geometrical value. In fact, deviations from the certified geometrical values for most of the lines considered during this work were less than 5 \(\mu\)m; this being of a similar order as the expected overall uncertainty in the geometrical length values.

![Figure 4](image)

The next step in testing this method is a bilateral comparison (by exchanging some of the air lines) to check the consistency of our results and to determine a realistic uncertainty budget. The outcome will be presented at the conference.

**VI. Conclusion**

The method for measuring resistivity is sound. The subsequent agreement between electrical and geometrical length determinations promises a realistic path to traceability for phase measurements, e.g. made using VNAs.

**VII. References**


