Traceability to National Standards for S-parameter Measurements of Waveguide Devices from 110 GHz to 170 GHz

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Abstract—This paper describes a new facility that has been introduced recently to provide high precision traceable scattering coefficient measurements of waveguide devices in the frequency range 110 GHz to 170 GHz (i.e. in waveguide size WR-06). The facility comprises measurement instrumentation situated at the University of Leeds and associated primary reference standards provided by the National Physical Laboratory. The instrumentation consists of a Vector Network Analyzer (VNA) and the standards are precision sections of waveguide that are used to calibrate the VNA. Traceability to national standards and the International System of units (SI) is achieved via precision dimensional measurements of the waveguide sections. Typical measurements, with uncertainties, are given to illustrate the current state-of-the-art for traceable measurements of this type.

Index Terms—Vector network analysis, calibration and measurement, waveguides, millimeter-waves, traceability to national standards.

I. INTRODUCTION

For many years, leading National Metrology Institutes (NMIs) from around the world have offered capabilities that provide high precision reflection and transmission coefficient measurements in various transmission media at RF and microwave frequencies. These capabilities establish traceability to the international system of units (SI) for these measurements by linking the measurands to the SI base units (i.e. the metre, ampere, second, etc [1]) in an efficient manner. As such, these systems provide the national and international references for such measurements and ensure harmonisation of all measurements that link to these references.

To date, all these NMI capabilities fall within a frequency range from a few kHz to 110 GHz. Above 110 GHz, no such national measurement systems have been available for ensuring the quality of measurements made at these frequencies. This frequency limit corresponds with the upper frequency limit of current precision coaxial connector technology (i.e. the 1 mm connector [2]). It also corresponds with the upper limit of the smallest commonly used rectangular waveguide size (i.e. WR-10 [3]). However, in recent years, instrumentation manufacturers, in response to demands coming from applications in the industry (see, for example, [4-9]), have begun to offer measurements systems, i.e. Vector Network Analyzers (VNAs), that operate at frequencies above this 110 GHz ‘boundary’ (see, for example, [10-13]). This, in turn, is driving the need for reliable references and quality assurance mechanisms for measurements at these frequencies.

In response to this need, the UK has begun a program of research to put in place national reference standards and measurement capabilities that provide the level of accuracy that is required, along with traceability of measurements to SI. This program is being delivered through a partnership between the University of Leeds and the National Physical Laboratory (NPL). Essentially, equipment situated in the research laboratories at the University of Leeds is calibrated using standards supplied by NPL. The equipment comprises a millimeter-wave VNA and the calibration standards are sections of high precision waveguide. Staff at both the University of Leeds and NPL are involved in providing this measurement capability.

To date, work has concentrated on the WR-06 waveguide size that enables measurements to be made from typically 110 GHz to 170 GHz [3]. This paper describes the new national standard measurement facility that has been put in place to provide these traceable reflection and transmission coefficient measurements (i.e. S-parameters) in this frequency range.
II. PIMMS

The measurement method used by this facility is an extension of NPL’s Primary Impedance Measurement System (PIMMS) [14-16]. This system uses a commercially available VNA as the measuring instrument along with control software running on an external PC. The PC is linked to the VNA via a GPIB, or similar, connection. The software sends and receives information to and from the VNA (as GPIB commands and uncorrected voltage ratios, respectively); provides instructions for the VNA operator (for connecting devices during both calibration and measurement); performs the necessary calculations to implement the VNA calibration and measurement algorithms; and, processes data so that the overall uncertainty of measurement can be established.

The measurement instrument comprises an Agilent Technologies 8510C VNA and a set of Oleson Microwave Labs millimeter-wave extension modules. A pair of millimeter-wave modules forms a complete $S$-parameter test set operating over a particular waveguide band. For the WR-06 waveguide band, the instrument operates with a nominal test port power of −20 dBm, which is obtained via harmonic multipliers within each extension module. The University of Leeds system employs an Agilent 8360 synthesized source operating from 18 GHz to 29 GHz to drive the ×6 multipliers for the RF test signal. Harmonic mixers are used to down-convert the Test and Reference signals for the receiver using a second (phase locked) synthesized source as the Local Oscillator. The OML millimeter-wave modules are connected to the 8510C via a dedicated controller (Agilent 85105A-K10) which manages the routing and conditioning of the RF, LO and IF signals – see Figure 1. System dynamic range is typically around 70 dB across the WR-06 waveguide band.

The uncertainty in the VNA measurements is evaluated in accordance with international guidelines [17] with modifications to account for the measurands (i.e. the $S$-parameters) being complex-valued quantities [18]. The measurement strategy makes use of multiple repeated connections to enable the size of the random errors affecting the measurements to be determined. The size of the systematic errors in the measurement process (e.g. due to imperfections in the physical properties of the calibration standards, VNA detectors’ non-linearity, cross-talk, etc) is established by performing separate experiments. The evaluation of uncertainty is described in detail in section VI.

III. CALIBRATION STANDARDS AND TECHNIQUES

PIMMS provides the UK’s national reference for $S$-parameter measurements. It therefore strives to achieve the best accuracy from available calibration standards, calibration techniques and instrument hardware. Since uniform sections of transmission line are fundamental to our understanding of microwave circuit theory, the physical realization of such lines (e.g. as precisely machined sections of air-filled waveguide) makes an ideal choice of reference standard for these measurements.

NPL has acquired a pair of these precise waveguide sections in the WR-06 waveguide size [3] for use with PIMMS. The calibration techniques supported by PIMMS are Thru-Reflect-Line (TRL) [19] and Line-Reflect-Line (LRL) [20]. These techniques have been chosen since they make optimum use of lines as standards for the measurement process. TRL and LRL are also “self-calibration techniques” that do not require all standards to be fully known [21]. It is only the first standard (i.e. the Thru in TRL, and the first Line standard in LRL) that is required to be fully known (i.e. it is assumed that all four $S$-parameters of this standard are known) during the calibration process. This is generally a safe assumption to make for a Thru connection, since the properties of the Thru do not need to

Figure 1: Photograph showing the WR-06 VNA system at the University of Leeds
determined by measurement. However, for the properties of a Line to be fully known, the line must first be characterised using dimensional measurements (particularly, to determine the length of the Line standard). The inevitable errors in the dimensional measurements will lead to errors in the assumed known characteristics of the Line standard. At frequencies well below 110 GHz (where wavelengths are comparatively large), it is safe to assume that such dimensional errors will be insignificant compared with the accuracy required to achieve a satisfactory characterisation of the waveguide Line standard. However, at higher frequencies (e.g. above 110 GHz), where wavelengths are comparatively small, this assumption becomes more difficult to sustain. This is because the dimensional errors will become a significant fraction of the wavelength and hence the waveguide Line standard cannot be defined well enough to be a known standard in the calibration scheme.\footnote{The length of the Line standard translates directly into the assumed phase of the transmission coefficients, \( S_{11} \) and \( S_{21} \), for the line. A length error therefore translates directly into a transmission coefficient phase error. A length error of constant value will produce a phase error that increases with frequency (i.e. with decreasing wavelength). Therefore, such an error is likely to be insignificant at low frequencies but increasingly significant at higher frequencies.}

For this reason, it has been decided not to use the LRL calibration technique with PIMMS at these higher millimeter-wave frequencies. Instead, a version of TRL has been developed that makes use of phase changes that are greater than those usually used with conventional TRL. In a conventional TRL calibration, the length of the Line standard is chosen so that the phase difference between the Thru connection and the Line connection falls well within the 0º to 180º region across the full range of frequencies for the waveguide size. This is because the calibration performs well for phase changes within this range, but fails completely at 0º and 180º. In fact, in general, the calibration fails when the phase change is \( n \times 180º \) (where \( n = 0, 1, 2, \ldots \)). Therefore, to ensure reliable and robust calibrations, the line length is chosen so that the phase difference between the Thru and Line connections falls well within the 0º to 180º region across the full range of frequencies for the waveguide size. It becomes more difficult to meet this criterion using TRL calibration schemes at these high frequencies because the length of line needed becomes quite short. For example, in WR06, from 110 GHz to 170 GHz, a line of length 0.73 mm will provide phase changes that fall within this criterion. However, such a short section of line could easily bend or become distorted during use and so is not a practical choice as a standard at these frequencies.

To avoid using such short sections of line, longer lengths of line are chosen to produce phase changes that fall well within the 180º to 360º range, i.e. 30º, or more, away from these calibration failure points. This means that phase changes that lie within (180º + 30º) = 210º and (360º – 30º) = 330º will provide acceptable TRL calibrations. However, these longer lengths of line produce phase changes that vary more rapidly with frequency and so a single line is not sufficient to cover the full waveguide band. Instead, two lines are used – one each to cover the lower and upper parts of the waveguide band. This approach to TRL calibration at higher millimeter-wave frequencies has been described in detail in [22]. The approach leads to the choice of line lengths for the WR-06 waveguide size given in Table 1.

### Table 1: Line lengths for TRL calibration in the WR-06 waveguide band

<table>
<thead>
<tr>
<th>Line length (mm)</th>
<th>Frequency range (GHz)</th>
<th>Phase change (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>Maximum</td>
<td>Minimum</td>
</tr>
<tr>
<td>2.81</td>
<td>110</td>
<td>133</td>
</tr>
<tr>
<td>1.91</td>
<td>129</td>
<td>170</td>
</tr>
</tbody>
</table>

Notice that there is an overlap between the bandwidths achieved by the two lines in Table 1. The 2.81 mm line has a usable upper frequency limit of 133 GHz whereas the 1.91 mm line has a usable lower frequency limit of 129 GHz. This means that the changeover between lines during calibration can occur anywhere from 129 GHz to 133 GHz. In practice, this changeover frequency is chosen to be 131 GHz.

Figure 2 shows a photograph of the two Line standards from the PIMMS TRL calibration kit. This photo shows that each line includes a metal ‘tab’ that allows the standard to be handled easily without touching the waveguide flange faces.
The TRL calibration technique also uses a Reflect standard that is required to provide the same, but not necessarily known, value of reflection at each of the VNA’s test ports. In practice, the Reflect standard is realized using a flush short-circuit which is connected, in turn, to both VNA test port reference planes. This is therefore assumed to provide the required same value of reflection at both of the VNA’s reference planes (neglecting errors due to connection repeatability and noise).

IV. DIMENSIONAL DATA

Traceability to SI is established for the S-parameter measurements via dimensional measurements of the waveguide Line standards. Specifically, measurements of the broad and narrow wall dimensions of the apertures of the waveguides are made using a Zeiss F25 coordinate measuring machine (CMM) fitted with a 0.3 mm diameter ball tip micro-stylus. The lines are measured individually, and are positioned on the CMM with the aperture axis aligned vertically, as shown in Figure 3.

During measurement a local coordinate system is set up on the line, with the origin being set as one corner of the rectangular aperture, one long edge of the aperture set as the x-axis and with z = 0 being the top surface of the line. The broad and narrow wall dimensions of the rectangular aperture are measured at a series of approximately equi-spaced locations across the aperture. The broad wall dimension is measured at 3 locations while the narrow wall dimension is measured at 5 locations. This 2D grid of measurements is repeated at 3 different depths inside the aperture. Each measurement run therefore produces 24 dimensional measurements, 9 width values and 15 height values.

The stylus of the CMM is too short to make measurements over the entire length of the line in one operation. It is therefore necessary to measure each line in 2 stages with the line flipped 180° between measurements, and with a set of overlap aperture size data taken at the nominal mid-plane of the line. Each measurement run is repeated 5 times, with the mean of the measurement data reported. The measurements are temperature corrected using a value $16.6 \times 10^{-6}$ K$^{-1}$ for the coefficient of linear thermal expansion of copper.

The measurement uncertainty of the reported dimensions is determined using a calibrated standard [23]. A stack of three reference gauge blocks is wrung together in such a way as to create a small channel with the same dimension as the nominal dimensions of the waveguide aperture. The width of the channel is measured at a series of locations and the mean dimension recorded. Using the principle described in [23] the measurement uncertainty of the reported dimensions is evaluated to be 0.390 µm ($k = 2$).

V. ELECTRICAL CHARACTERISATION

The nominal values for the broad and narrow wall dimensions of the aperture of WR-06 waveguide are 1.651 mm and 0.826 mm, respectively [3]. The measurements described in Section IV show that the apertures of the two lines used for the TRL calibration exhibit measurable departures from these nominal values. These measured values can be summarized in terms of the maximum observed deviation from the nominal dimensions of the waveguide. These summary values are shown in Table 2.
TABLE 2: Summary of dimensional measurements for the two TRL Line standards

<table>
<thead>
<tr>
<th>Waveguide Line nominal length (mm)</th>
<th>Maximum broad wall deviation (µm)</th>
<th>Maximum narrow wall deviation (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.81</td>
<td>8.1</td>
<td>7.8</td>
</tr>
<tr>
<td>1.91</td>
<td>14.9</td>
<td>13.0</td>
</tr>
</tbody>
</table>

In order to understand how the deviations in the nominal values impact on the calibration of the VNA, and subsequent accuracy of the calibrated VNA, it is necessary to convert these dimensional deviations into their equivalent electrical form. Specifically, to a first order approximation, to estimate the amount of reflection that will be generated when these lines are connected to lines with notionally perfect dimensions. In order to do this, the 3D electromagnetic simulation software CST Microwave Studio [24] is employed.

The magnitude of the reflection coefficient at the transition between two semi-infinite sections of lossless uniform waveguide is computed. One of the waveguide sections has a rectangular cross-section with internal dimensions equal to the nominal values for WR-06; the other section has a rectangular cross-section with dimensions which are either undersized or oversized compared to nominal by amounts equal to the maximum deviations measured for the NPL TRL line standards (i.e. 14.9 µm for the broad wall dimension and 13.0 µm for the narrow wall dimension – see Table 2). The two sections of waveguide are fitted together in a symmetrical way and the gaps between the two waveguides are closed by perfectly conducting transverse walls in the plane of the transition. The waveguide sections are not rotated with respect to one another so that corresponding walls in the two sections are parallel. A cross-sectional view of the transition used in the calculations is shown in Figure 4. Four reflection coefficients are calculated – with the broad wall and narrow wall dimensions of the second waveguide section separately made undersized and oversized. In each case, the linear magnitude of the reflection coefficient is obtained as a function of frequency from 110 GHz to 170 GHz. The reflection coefficients obtained for the transition to undersized waveguide are shown in Figure 5 with similar results being obtained for oversized waveguide (not shown).

The transient solver in CST Microwave Studio is used for these calculations. The solver performs its computations in the time domain using a pulse excitation and then converts the results into the frequency domain by means of a Fourier Transform.

In this way, the worst-case reflection coefficient magnitude due to deviation in the broad wall dimension, \(a\), is found to be \(|\Gamma_a| = 0.010\) and the worst-case reflection coefficient magnitude due to deviation in the narrow wall dimension, \(b\), is found to be \(|\Gamma_b| = 0.0080\).

![Figure 4: Cross-section of the waveguide transition used in the reflection coefficient calculations. The amount of oversize has been exaggerated. The transition to an undersized imperfect waveguide is similar.](image)

![Figure 5: Magnitude of the reflection coefficients due to a broad wall dimension, \(a\), undersized by 14.9 µm and a narrow wall dimension, \(b\), undersized by 13.0 µm.](image)

VI. UNCERTAINTY ESTIMATES

In order to specify the performance of the VNA measurement system, uncertainty budgets are constructed showing the likely size of uncertainty contributions due to systematic errors in the system (e.g. residual terms in the error model, crosstalk/isolation, non-linearity, etc). These uncertainty budgets do not include contributions due to random errors (e.g. flange connection repeatability, noise and ambient...
conditions). These estimates therefore provide a Best Measurement Capability (BMC) suitable for defining a Scope of Accreditation for this measurement system.

A. Reflection measurements

In the case of reflection measurements (i.e. \(S_{11}\) and \(S_{22}\)), the principal source of uncertainty will be due to reflections caused by the Line standards used during calibration. Section V gave values for the reflections caused by the maximum deviations in the broad wall, \(a\), and narrow wall, \(b\), dimensions of the Line standards. Since these \(\Gamma_a\) and \(\Gamma_b\) values are effectively worst-case estimates, they can be treated as limit values and can be characterised using uniform distributions. Their equivalent standard uncertainty, \(u(\Gamma_a)\) and \(u(\Gamma_b)\), can therefore be established in the usual way [17]:

\[
u(\Gamma_a) = \frac{|\Gamma_a|}{\sqrt{3}} \quad \text{and} \quad \nu(\Gamma_b) = \frac{|\Gamma_b|}{\sqrt{3}} \quad (1)
\]

Also, since the determinations of \(\Gamma_a\) and \(\Gamma_b\) are essentially independent of each other, they can be combined in the usual way [17], to give an overall combined standard uncertainty, \(u(\Gamma)\), due to dimensional deviations in the Line standards:

\[
u(\Gamma) = \sqrt{u(\Gamma_a)^2 + u(\Gamma_b)^2} \quad (2)
\]

We now use the values of \(\Gamma_a\) and \(\Gamma_b\) determined in Section V, along with equations (1) and (2), to provide our determination of \(u(\Gamma)\):

\[
u(\Gamma) \approx 0.0074
\]

Therefore, this value can be considered the BMC standard uncertainty for reflection coefficient measurements (i.e. \(|S_{11}|\) and \(|S_{22}|\)). It can be considered as equivalent to an estimate of the residual directivity in the calibrated VNA, this being the major systematic error for measurements of low values of reflection.

The expanded uncertainty in reflection coefficient measurements, \(U(\Gamma)\), is given by [17]:

\[
U(\Gamma) = 2 \times u(\Gamma) = 0.015
\]

B. Transmission measurements

For transmission measurements (\(S_{21}\) and \(S_{12}\)), the model given in [27] is used to determine the overall uncertainty. The three principle components are:

(i) isolation/crosstalk;
(ii) mismatch;
(iii) non-linearity.

(i) The isolation/crosstalk is determined by observing \(|S_{21}|\) and \(|S_{12}|\) when both ports of the VNA are terminated with low reflecting loads. This is shown in Figure 6:

![Figure 6: Isolation assessment for the VNA with both ports terminated with one-port devices](image)

Figure 6 shows that the isolation error, \(I\), at all frequencies across the waveguide band is better than –60 dB. During measurement of a particular device under test, the size of the uncertainty contribution due to isolation/cross-talk, \(dA\), will vary depending on the measured attenuation, \(A\), according to [27]:

\[
dA = 20 \log_{10} \left[ \frac{I + A}{1 + 10^{20}} \right] \quad (3)
\]

\[\text{Except for noise, these random errors come from 'outside' the VNA and so are not representative of the VNA’s performance. Flange connection repeatability errors are due to the quality of the flanges on the devices under test. Effects due to ambient conditions are minimised by keeping the time between VNA calibration and subsequent measurement of devices as short as possible. In addition, the VNA is located in a temperature-controlled laboratory.}\]
Since \(dA\) is effectively a worst-case value, it can be treated as a limit value and can be characterised using a uniform distribution. The equivalent standard uncertainty, \(u(dA)\), can therefore be established in the usual way [17]:

\[
u(dA) = \frac{dA}{\sqrt{3}} \quad (4)
\]

(ii) The expression used to evaluate the mismatch error, \(M_{TM}\), is [27]:

\[
M_{TM} = 20 \log_{10} \left\{ 1 + \left( |M S_{11}| + |\Gamma L S_{12}| + |M T S_{21}| + |M T S_{22}| \right) \right\} \frac{\left(1 - |M T| \right)}{\left(1 - |\Gamma L| \right)}
\]

where \(M\) is the residual test port match, \(\Gamma L\) is the residual load match and \(S_{11}, S_{22}, S_{21}\) and \(S_{12}\) are the \(S\)-parameters of the device under test.

It is assumed here that \(M\) and \(\Gamma L\) can be taken as equal to the value for the standard uncertainty for measuring low values of reflection, derived above (i.e. 0.007 4). Also, for convenience, we will restrict the estimate of mismatch to consider only devices with relatively low input and output reflection, i.e. where \(|S_{11}| \leq 0.1\) and \(|S_{22}| \leq 0.1\). Under these conditions, the worst-case value of \(M_{TM}\) is 0.013 8 dB (regardless of the values of \(S_{21}\) and \(S_{12}\)).

As is conventional for vector errors where knowledge of the phase of the vector is absent, a \(U\)-shaped distribution is assigned to this error and therefore the equivalent standard uncertainty, \(u(M_{TM})\), can be established as [28, 29]:

\[
u(M_{TM}) = \frac{M_{TM}}{\sqrt{2}} \quad (6)
\]

(iii) The non-linearity, \(L\), in the VNA’s transmission measurements can be assessed by measuring a calibrated change in attenuation at different input power levels. However, such a calibrated attenuation ‘step’ is not yet available and so published values for \(L\) are used to provide a guide estimate for this uncertainty contribution. The value for \(L\), given in [27] for a coaxial VNA system, is 0.002 dB/dB. Therefore a somewhat conservative value of 0.004 dB/dB is used here for uncertainty budgeting purposes.

Equations (4), (6) and (7) are used to determine the overall uncertainty for transmission measurements, expressed in dB (i.e. attenuation measurements). The combined standard uncertainty for attenuation measurements, \(u(A)\), is given by:

\[
u(A) = \sqrt{\left( u(dA) \right)^2 + \left( u(M_{TM}) \right)^2 + \left( u(L) \right)^2} \quad (8)
\]

Therefore, this value can be considered the BMC standard uncertainty for attenuation measurements (i.e. \(|S_{21}|\) and \(|S_{12}|\). The expanded uncertainty in attenuation measurements, \(U(A)\), is given by [17]:

\[
U(A) = 2 \times \nu(A) \quad (9)
\]

Both \(u(A)\) and \(U(A)\) will vary considerably depending on the value of attenuation being measured. It is therefore informative to examine the size of each contributing uncertainty component (isolation/crosstalk, mismatch, non-linearity) as a function of measured attenuation. This is shown in Figure 7 for measured values of attenuation ranging from 0 dB to 30 dB. This figure shows that, for low values of attenuation (i.e. 5 dB and less), mismatch is the main source of uncertainty; for values of attenuation ranging from 5 dB to 15 dB, non-linearity is the main source of uncertainty; and, for high values of attenuation (i.e. greater than 15 dB), isolation is the main source of uncertainty. The resulting overall expanded uncertainty (using equations (8) and (9)), shown in Figure 8, varies from 0.02 dB to 0.34 dB as the measured attenuation varies from 0 dB to 30 dB.

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Figure 7: showing the standard uncertainty for uncertainty components of attenuation measurements

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5 For devices where \(|S_{11}|\) and \(|S_{22}|\) are greater than 0.1, these calculations need to be repeated using the measured values of \(|S_{11}|\) and \(|S_{22}|\) in the above equation.
This uncertainty information can also be summarized in the form of an uncertainty budget, as shown in Table 3.

In practice, the calculation of uncertainty is performed at each frequency and at each measured value. This can lead to values of uncertainty that are considerably lower than the values shown in Figures 7 and 8, and determined in Table 3. For example, it is clear from Figure 6 that, at most frequencies, the VNA’s isolation is much better than the -60 dB that is used to estimate the overall uncertainty of the measurements.

C. Uncertainty in phase

For a given S-parameter, $S_{ij}$ ($i = 1, 2; j = 1, 2$), the expanded uncertainty in phase, $U(\phi)$, can be estimated using [30]:

$$U(\phi) = \sin^{-1}\left(\frac{U|S_{ij}|}{|S_{ij}|}\right)$$

where $S_{ij}$ is the measured S-parameter and $U(|S_{ij}|)$ is the expanded uncertainty in $|S_{ij}|$.

When calculating the uncertainty in transmission phase, it is first necessary to determine the uncertainty in the magnitude of the linear transmission coefficient (i.e. $U(|S_{21}|)$ or $U(|S_{12}|)$). This can be derived from the measured attenuation, $A$, and the uncertainty in the measured attenuation, $U(A)$, using [31]:

$$U(|S_{ij}|) = \frac{1}{8.686} \times 10^{-20} \times \frac{A}{U(A)}$$

Equation (10) shows that the uncertainty in the phase of a given S-parameter will depend strongly on the magnitude of the S-parameter. For magnitudes close to unity, the uncertainty can be approximated by:

$$U(\phi) = \sin^{-1}\left(\frac{U(|S_{ij}|)}{|S_{ij}|}\right)$$

Equation (12) can be used to establish a Best Measurement Capability for S-parameter reflection and transmission phase measurements. However, when the magnitude of the S-parameter becomes of a similar size to the uncertainty in the magnitude, the uncertainty in phase becomes indeterminate. As an example of a phase uncertainty calculation, the uncertainty budget shown in Table 3 for the $S_{21}$ measurement of a 20 dB attenuator gives an expanded uncertainty of 0.13 dB. This is equivalent to an uncertainty in $|S_{21}|$, using equation (11), of 0.0015. This leads to an uncertainty in the phase of $S_{21}$, using equation (10), of 0.86º.

### TABLE 3: Uncertainty budget for an $S_{21}$ measurement, assuming $|S_{11}| = |S_{22}| = 0.1$ and $|S_{21}| = |S_{12}| = 0.1$ (i.e. a well-matched 20 dB attenuator)

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Estimate</th>
<th>Uncertainty (dB)</th>
<th>Distribution</th>
<th>Divisor</th>
<th>Uncertainty (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>0.004 dB/dB</td>
<td>0.080 0 dB</td>
<td>Gaussian</td>
<td>2</td>
<td>0.040</td>
</tr>
<tr>
<td>Mismatch</td>
<td>0.013 8 dB</td>
<td>0.086 4 dB</td>
<td>U-shaped</td>
<td>$\sqrt{2}$</td>
<td>0.009</td>
</tr>
<tr>
<td>Isolation/crosstalk</td>
<td>-60 dB</td>
<td>0.086 4 dB</td>
<td>Rectangular</td>
<td>$\sqrt{3}$</td>
<td>0.050</td>
</tr>
<tr>
<td>Combined standard uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.065</td>
</tr>
<tr>
<td>Expanded uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.13</td>
</tr>
</tbody>
</table>
VII. SUMMARY

This paper has described a new facility that has recently been put in place for providing precision, traceable, S-parameter measurements of waveguide devices in the frequency range 110 GHz to 170 GHz. The facility has been realized through a partnership between the University of Leeds and NPL. In particular: the instrumentation – the VNA system – is owned by, and operated at, the University of Leeds; the primary reference standards – the TRL calibration kit – is owned by NPL; the measurement software is provided by NPL; scientists at both the University of Leeds and NPL are involved in operating the service.

It has been shown that this measurement system achieves a Best Measurement Capability (BMC) uncertainty of 0.015 when measuring low values of linear reflection coefficient magnitude, and 0.02 dB when measuring low values of attenuation. The associated BMC for phase measurements, using equation (12), is 0.86º for reflection measurements and 0.13º for transmission measurements.

Since this service essentially represents a ‘distributed’ measurement system, with system ‘components’ coming from two different geographical locations, a longer term objective for operating this system is to utilize the Internet to drive the measurement process. Such an approach is already being used successfully for S-parameter measurements at lower microwave frequencies [32].

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6 The BMC uncertainties are Expanded Uncertainties [17] at an estimated level of confidence of 95%.


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