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Evaluating and expressing uncertainty in high-frequency electromagnetic measurements: a selective review

Nick M Ridler and Martin J Salter
National Physical Laboratory, Teddington, UK
E-mail: nick.ridler@npl.co.uk
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Abstract
This paper provides a selected review of topics relating to evaluating and expressing uncertainty for some measurands that occur in high-frequency electromagnetic metrology. Specific emphasis is given to complex-valued quantities (i.e. vector measurands having both an associated magnitude and phase component), such as scattering parameters (i.e. S-parameters) used at radio, microwave, millimetre-wave and terahertz frequencies.

Keywords: complex-valued quantities, electromagnetic measurement, microwave S-parameters, uncertainty of measurement
(Some figures may appear in colour only in the online journal)

1. Introduction
Science, engineering and technology employing the use of electrical signals has been in existence for a very long time—i.e. more than a century. Throughout this period, measurements have been required to demonstrate and validate new and evolving technologies. At all stages of technological development—from research, through development, to production and testing—measurements are needed to verify the technology that is being developed. In addition, knowledge about the uncertainty in these measurements has been necessary to help understand the reliability of the measurements (recognizing that the measurements themselves have often been very challenging to make with confidence).

An outcome from this process has been a long-standing awareness (going back over many decades) of the importance of establishing the uncertainty in measurements made of electrical quantities. However, a review of the literature over this period shows that a variety of methods were used to determine and express the uncertainty in the measurements. These different methods could often give rise to different values, and different forms, of uncertainty being specified for effectively the same type of measurement result. This lack of a ‘standardized’, agreed, approach to evaluating and expressing the uncertainty in measurement gave rise to much debate at all levels of the metrological hierarchy—from the manufacturing shop-floor, through accredited calibration laboratories, and ultimately to the national measurement laboratories. Eventually, this situation was recognized by the world’s highest authority in metrology, the Comité International des Poids et Mesures (CIPM), who requested that the Bureau International des Poids et Mesures (BIPM) address the problem in conjunction with the national measurement laboratories. This led to the setting up of an International Working Group that subsequently developed a recommendation on the expression of experimental uncertainties [1].

It is this recommendation that led subsequently to the development and publication of the Guide to the Expression of Uncertainty in Measurement (GUM) [2]. The GUM has since been supplemented by further documents [3–6] covering various aspects relating to uncertainty in measurement. This on-going series of documents is still being added to [7–9].

This GUM ‘family’ of documents has made a major impact on the field of electrical metrology. For example, since many of the measurands encountered in electrical metrology are complex-valued quantities (having magnitude and phase, or, equivalently, both real and imaginary components) the covariance approach, advocated in the GUM [2] (for determining combined standard uncertainty for correlated input quantities) has become the method of choice for evaluating uncertainty in many electrical measurement situations. In addition, the prevalence of nonlinear measurement models has meant that the techniques
presented in GUM supplement 1 [4] have been very useful for dealing with effects due to these nonlinear models. Finally, it is very common in electrical metrology for the measurement model to have both multiple input and multiple output quantities (with many of these quantities often being complex-valued quantities). In these circumstances, the techniques presented in GUM supplement 2 [5] have been very useful. In short, the global electrical metrology community has benefitted greatly from the work that has been done (and is continuing to be done) in establishing and maintaining this GUM family of documents.

This paper reviews just a few instances where the approaches advocated in the GUM family of documents have been applied to electrical metrology1. Specific emphasis is given to situations encountered in high-frequency electromagnetic metrology since these situations exhibit the properties described above—complex-valued quantities, nonlinear measurement models, and multiple input and output quantities.

2. Representing complex-valued measurands

Electrical signals at high frequencies can be considered as waveforms that vary continuously with time. The most obvious (and simplest) type of time-dependent variation can be represented using a sine wave. Most continuously varying electrical signals can be ‘decomposed’ into a series of sine waves, where each sine wave will have both a magnitude and phase component. Engineers and scientists that are involved in electromagnetic technology tend to ‘think’ in terms of waves, where each sine wave will have both a magnitude and phase component. Instead, the distribution is skewed with an average value greater than the expected value of zero—hence, |S| > 0 when |S| = 0. Instead, the distribution is skewed with an average value greater than the expected value of zero—hence, |S| > 0 when |S| = 0.

A similar problem occurs when calculating the experimental standard deviation of |S|, s(|S|), using:

\[ s(|S|) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (|S|_i - \overline{|S|})^2}. \]

In this case, the calculation of the experimental standard deviation under-estimates the expected, underlying, standard deviation. Both these effects have been discussed previously in [10].

Calculations performed on phase measurements can also present some difficulties: the cyclical nature of the phase scale (either ranging from −180° to +180° or from 0° to 360°) requires a form of ‘clock’ arithmetic to be used for these calculations. For example, averaging phase values +179° and −177° using conventional arithmetic leads to a result of +1°, since:

\[ \frac{+179° + (-177°)}{2} = +1° \]

whereas the expected result, on a scale ranging from −180° to +180°, is −179°. These effects have also been discussed previously in [10].

Due to the above-mentioned difficulties, the following approach for statistically analysing complex-valued (vector) data has been developed [10]:

Both these types of measurement scale can cause problems when performing statistical calculations on the repeated vector data. For example, for vectors with small magnitudes, a systematic error can be introduced if the magnitude values are averaged, as illustrated below.

A complex-valued measurand, S, can be represented in terms of its real and imaginary components, S_R and S_I, respectively, as:

\[ S = S_R + jS_I \]

where \( j^2 = -1 \).

For a series of n repeated determinations of S, the mean magnitude, \( \overline{|S|} \), is calculated using:

\[ \overline{|S|} = \frac{1}{n} \sum_{i=1}^{n} |S|_i \]

where \( |S|_i \) represents the magnitude of the i\(^{th} \) measurement of S.

For a device with \(|S| \approx 0\), every measured \(|S|_i\) will always be greater than zero (but never less than zero) due to random errors affecting the measurement process. The repeated complex-valued measurements of S, when displayed in the complex plane, appear as a two-dimensional (i.e. bivariate) normal distribution centred on the origin of the complex plane. However, the distribution of \(|S|\) is not (and cannot be) normally distributed and centred on the expected value of \(|S| = 0\). Instead, the distribution is skewed with an average value greater than the expected value of zero—hence, \(|S| > 0\) when \(|S| = 0\).

1 The paper is based on an invited presentation that was given at the conference ‘Guide to the Expression of Uncertainty in Measurement: Past, Present and Future’, which took place at the National Physical Laboratory, UK, in November 2013. The conference marked the 20th anniversary of the first publication of [2].

2 We use here the symbol, S, to represent a complex-valued quantity as it is also the symbol that is used to denote electromagnetic scattering parameters (i.e. S-parameters) which are referred to later in this paper.
and conveniently using an uncertainty matrix. For our complex-valued quantity, the uncertainty in a vector quantity can be expressed using an uncertainty matrix. This assumes the data is drawn from a bivariate normal distribution. An example uncertainty ellipse is shown in figure 1.

When the elements in the uncertainty matrix have been derived from statistical evaluations, we can assume (from equations (7) and (8)):

\[
{\bar{s}} (s) = s(\bar{s})
\]

and

\[
{\bar{u}} (s) = \frac{s}{s(\bar{s})}.
\]

The off-diagonal elements in the uncertainty matrix, \( {\bar{u}}(\bar{s}_R, \bar{s}_I) \) and \( {\bar{u}}(\bar{s}_I, \bar{s}_R) \), are related to the covariance, \( s(\bar{s}_R, \bar{s}_I) \), of \( \bar{s}_R \) and \( \bar{s}_I \), which is given by

\[
s (\bar{s}_R, \bar{s}_I) = \frac{1}{n(n-1)} \sum_{i=1}^{n} (\bar{s}_{Ri} - \bar{s}_R)(\bar{s}_{Ii} - \bar{s}_I)
\]

so

\[
{\bar{u}} (\bar{s}_R, \bar{s}_I) = u (\bar{s}_I, \bar{s}_R) = s (\bar{s}_R, \bar{s}_I).
\]

Alternatively, the correlation coefficient, \( \rho (\bar{s}_R, \bar{s}_I) \), can be calculated to indicate the degree of correlation between \( \bar{s}_R \) and \( \bar{s}_I \)

\[
\rho (\bar{s}_R, \bar{s}_I) = \frac{u (\bar{s}_R, \bar{s}_I)}{u (\bar{s}_R) u (\bar{s}_I)}.
\]

There are alternative approaches for dealing with these difficulties—see, for example, \[11\].

### 3. Uncertainty matrices and correlation coefficients

The uncertainty in a vector quantity can be expressed conveniently using an uncertainty matrix. For our complex-valued quantity, \( S \), the uncertainty matrix is

\[
\begin{pmatrix}
{\bar{s}}_R & \bar{s}_I
\end{pmatrix}
\]

When the elements in the uncertainty matrix have been derived from statistical evaluations, we can assume (from equations (7) and (8)):

\[
{\bar{u}} (s) = s(\bar{s})
\]

and

\[
{\bar{u}} (s) = \frac{s}{s(\bar{s})}.
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\[
\sum_{i=1}^{n} (\bar{s}_{Ri} - \bar{s}_R)(\bar{s}_{Ii} - \bar{s}_I)
\]

so

\[
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Alternatively, the correlation coefficient, \( \rho (\bar{s}_R, \bar{s}_I) \), can be calculated to indicate the degree of correlation between \( \bar{s}_R \) and \( \bar{s}_I \)

\[
\rho (\bar{s}_R, \bar{s}_I) = \frac{u (\bar{s}_R, \bar{s}_I)}{u (\bar{s}_R) u (\bar{s}_I)}.
\]
Such a geometric representation of the uncertainty for a complex-valued quantity can be useful for observing and understanding trends in $S$-parameter uncertainties. For example, for high-frequency electromagnetic measurements, changes in the size, shape and orientation of the uncertainty ellipse can be examined as a function of the measurement frequency.

### 5. Using complex-valued uncertainties

In the majority of high-frequency electrical measurement situations, it is very common to measure simultaneously more than one complex-valued quantity. One such situation is the measurement of the complex-valued scattering parameters ($S$-parameters) of a high-frequency electronic component or circuit. Under these circumstances, a measuring instrument such as a vector network analyser (VNA) is used for the measurements. Most VNAs have two measurement ports and this enables a two-port circuit (e.g. a component with an input and an output) to be measured. The four complex-valued measurement parameters are: (i) reflection at the input; (ii) reflection at the output; (iii) transmission from input to output (i.e. forward transmission); (iv) transmission from output to input (i.e. reverse transmission). The measurement parameters are identified using subscript indices—$S_{11}$ and $S_{22}$ for the input and output reflections, respectively; and, $S_{12}$ and $S_{21}$ for the forward and reverse transmissions, respectively. These four $S$-parameters are usually arranged in a scattering matrix, as this representation is useful in implementing calculation strategies used in microwave circuit theory. So, for a two-port network:

\[
[S] = \begin{bmatrix}
S_{11} & S_{21} \\
S_{21} & S_{22}
\end{bmatrix}
\]  

(15)

The situation where a (vector) measurand (i.e. the output quantities) is determined from series of multiple complex-valued $S$-parameters (as the input quantities) can generally be represented as:

\[
w = f(S)
\]  

(16)

where $w$ is the vector of $m$ output quantities, $S$ is the scattering matrix for the multiple $n$ input complex-valued $S$-parameters, and $f$ is the function describing the measurement model.

A relatively simple example of this situation, which occurs regularly in microwave engineering, is when a single complex-valued vector measurand is determined from a single complex-valued $S$-parameter. The uncertainty in the complex-valued vector output quantity, $w$, can be obtained from the uncertainty in the complex-valued $S$-parameter input quantity using

\[
V_w = J V_S J^T
\]  

(17)

where $V_w$ is the $(2 \times 2)$ uncertainty matrix for $w$:

\[
V_w = \begin{bmatrix}
u^2(w_R) & u(w_R, w_I) \\
u(w_R, w_I) & u^2(w_I)
\end{bmatrix}
\]  

(18)

$V_S$ is the $(2 \times 2)$ uncertainty matrix for $S$:

\[
V_S = \begin{bmatrix}
u^2(S_R) & u(S_R, S_I) \\
u(S_R, S_I) & u^2(S_I)
\end{bmatrix}
\]  

(19)

and $J$ is the $(2 \times 2)$ Jacobian matrix that represents the sensitivity coefficients of the measurement model, $f$. ($J^T$ represents the matrix transpose of the Jacobian matrix.)

\[
J = \begin{bmatrix}
\frac{\partial f}{\partial S_R} & \frac{\partial f}{\partial S_I} \\
\frac{\partial f}{\partial S_I} & \frac{\partial f}{\partial S_I}
\end{bmatrix}
\]  

(20)

### 6. Transformation to impedance

The input impedance of a one-port electronic circuit component is often of interest e.g. in order to obtain a lumped element equivalent circuit model for the component. At microwave frequencies, rather than measuring the input impedance directly, it is usually more convenient to measure the voltage reflection coefficient (VRC, equivalent to the $S_{11}$ parameter, mentioned previously) using a VNA and then to deduce the impedance from the measured VRC. The impedance $Z$ and the VRC $\Gamma$ are both complex-valued quantities and the relationship between them is a bilinear
transformation i.e. a M"obius transformation. In particular, the measurement model for the situation under consideration is [12]:

\[ z = \frac{Z}{Z_0} = \frac{1 + \Gamma}{1 - \Gamma} \]  
(21)

where the VRC, \( \Gamma = p + jq \), is associated with the reference impedance, \( Z_0 \), and the normalized impedance is equal to \( z = \frac{Z}{Z_0} = r + jx \) where \( r \) and \( x \) are the normalized resistance and reactance respectively. Equation (21) is a particular case of equation (16) and is an example of a nonlinear measurement model. This complex-valued function of a complex variable has a singularity (simple pole) at \( \Gamma = 1 + j0 \) which corresponds to an open-circuit. The transformation can be thought of as a two-dimensional vector-valued function of a two-dimensional vector variable with component functions

\[ r = \frac{1 - p^2 - q^2}{(1 - p)^2 + q^2} \]  
(22)

and

\[ x = \frac{2q}{(1 - p)^2 + q^2}. \]  
(23)

The Jacobian matrix of the transformation is given by

\[ J = \begin{pmatrix} \frac{\partial r}{\partial p} & \frac{\partial r}{\partial q} \\ \frac{\partial x}{\partial p} & \frac{\partial x}{\partial q} \end{pmatrix} \]  
(24)

where the sensitivity coefficients, i.e. partial derivatives, are as follows

\[ \frac{\partial r}{\partial p} = \frac{2 (1 - p) - 2q^2}{[(1 - p)^2 + q^2]^2} \]  
(25)

\[ \frac{\partial r}{\partial q} = \frac{-4q (1 - p)}{[(1 - p)^2 + q^2]^2} \]  
(26)

\[ \frac{\partial x}{\partial p} = \frac{4q (1 - p)}{[(1 - p)^2 + q^2]^2} \]  
(27)

\[ \frac{\partial x}{\partial q} = \frac{2 (1 - p)^2 - 2q^2}{[(1 - p)^2 + q^2]^2}. \]  
(28)

Note that the partial derivatives and hence the Jacobian matrix are not defined at the singularity (occurring at \( p = 1 \), and \( q = 0 \)). The transformation maps the unit circle centred at the origin of the complex VRC plane to the imaginary axis in the complex normalized impedance plane. The inside of the unit circle in the VRC plane i.e. the unit disk in the VRC plane is mapped to the right hand half of the impedance plane, i.e. to points with positive normalized resistance, and the outside of the circle is mapped to the left hand half of the impedance plane, i.e. to points with negative normalized resistance. Passive circuit components correspond to points inside the unit disk in the VRC plane i.e. they have a VRC with magnitude less than or equal to one and a normalized resistance greater than or equal to zero.

If the uncertainty matrix in the measured VRC is \( V_\Gamma \), then using equation (17), the uncertainty matrix in the normalized impedance, \( V_z \), is given by

\[ V_z = J \cdot V_\Gamma \cdot J^\top \]  
(29)

where \( J \) is the Jacobian matrix of the measurement model given by equations (24)–(28). To illustrate the application of equation (29), the uncertainty in normalized resistance is plotted in figure 3 for VRC values inside the unit disk—it being assumed that the standard uncertainties in the real and imaginary components of VRC are given by \( u(p) = u(q) = 0.005 \) and the covariance \( u(p, q) = 0 \) at all points inside the unit disk. It can be seen from figure 3 that the uncertainty in normalized resistance rises dramatically in the vicinity of the singularity in the measurement model at \( \Gamma = 1 + j0 \). In the particular case considered in figure 3, the uncertainty ‘surface’ for normalized reactance is identical to that for normalized resistance and the covariance between resistance and reactance is zero.

Equation (29) for the uncertainty matrix of the normalized impedance is based on a linear approximation to the measurement model. For the measurement model under consideration, it turns out that this approximation is reasonably good for most points in the VRC plane except for those points in the vicinity of the singularity at \( \Gamma = 1 + j0 \). In order to investigate the applicability of equation (29), a Monte Carlo method can be used to propagate distributions through the measurement model as described in [4].

As an example, consider a short-circuit (\( \Gamma = -1 + j0 \)) with measured VRC given in table 2. A bivariate normal distribution with mean vector equal to the measured VRC and covariance matrix equal to the uncertainty matrix in the VRC is assigned to the VRC. A random sample of size 10 000 is generated from this distribution and each point in the sample is transformed according to the measurement model. The Monte Carlo samples for VRC and normalized impedance are illustrated in figures 4(a) and (b), respectively. The measured value of normalized impedance and the corresponding uncertainty matrix can be estimated from the mean vector and covariance matrix of the Monte Carlo sample of normalized impedance values. In this case, the Monte Carlo method confirms the applicability of equation (29) as both methods give the same uncertainty estimates for normalized impedance, as shown in table 3.

A Monte Carlo method can also be used to illustrate the behaviour of the measurement model at its singularity.
Table 2. Measured VRC for a short-circuit.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real component of VRC</td>
<td>−1.000</td>
</tr>
<tr>
<td>Imaginary component of VRC</td>
<td>0.000</td>
</tr>
<tr>
<td>Standard uncertainty in real component of VRC</td>
<td>0.005</td>
</tr>
<tr>
<td>Standard uncertainty in imaginary component of VRC</td>
<td>0.005</td>
</tr>
<tr>
<td>Covariance between real and imaginary components of VRC</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 3. Normalized impedance for the short-circuit obtained either by equation (29) or by the Monte Carlo method.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized resistance</td>
<td>0.0000</td>
</tr>
<tr>
<td>Normalized reactance</td>
<td>0.0000</td>
</tr>
<tr>
<td>Standard uncertainty in normalized resistance</td>
<td>0.0025</td>
</tr>
<tr>
<td>Standard uncertainty in normalized reactance</td>
<td>0.0025</td>
</tr>
<tr>
<td>Covariance between normalized resistance and reactance</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 4. Measured VRC of an open-circuit.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real component of VRC</td>
<td>1.000</td>
</tr>
<tr>
<td>Imaginary component of VRC</td>
<td>0.000</td>
</tr>
<tr>
<td>Standard uncertainty in real component of VRC</td>
<td>0.005</td>
</tr>
<tr>
<td>Standard uncertainty in imaginary component of VRC</td>
<td>0.005</td>
</tr>
<tr>
<td>Covariance between real and imaginary components of VRC</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Consider the measurement of an open-circuit given in table 4. Monte Carlo samples for VRC and normalized impedance of size 1 000 000 based on the assignment of a bivariate distribution to the VRC are shown in figures 5(a) and (b), respectively, and histograms of the distribution of normalized resistance, normalized reactance and magnitude of normalized impedance are plotted in figures 6(a)–(c), respectively. It can be seen in figure 5(b) that there is a circular region near to the origin where there is a low probability of finding the normalized impedance but there is also a large non-localized region where the normalized impedance is likely to occur. It is difficult to see how a useful uncertainty region for normalized impedance could be constructed in this case (considering the distributions presented in figures 5(b) and 6(a)–(c)). This suggests that for an open-circuit, the results of the measurement would be better expressed in terms of admittance \( Y \) equal to the reciprocal of impedance. This is because the measurement model for normalized admittance, \( y \), is a well behaved, i.e. analytic, function in a neighbourhood of the point \( 1 + \jmath 0 \) corresponding to an open-circuit in the complex VRC plane

\[
y = \frac{Y}{Y_0} = \frac{1 - \Gamma}{1 + \Gamma}
\]

where \( Y_0 = 1/Z_0 \) is the reference admittance associated with the VRC.

7. Summary

This paper has presented a selected review of topics relating to evaluating and expressing uncertainty in high-frequency electromagnetic measurements. The primary focus of this review has been scattering (\( S \)-) parameter measurements that are used to characterize components, networks and circuits encountered at radio, microwave, millimetre-wave and terahertz frequencies. These \( S \)-parameters underpin just about all measurements made at these frequencies and so impact all areas of science, engineering and technology that rely on these frequencies.

From a metrological perspective, \( S \)-parameters exhibit some interesting features: (i) they are complex-valued (i.e. vector) quantities; (ii) it is common to measure multiple \( S \)-parameters simultaneously during a measurement exercise; (iii) measurement models involving \( S \)-parameters very often exhibit some form of nonlinearity; (iv) many output quantities (i.e. other measurands) rely on \( S \)-parameters as input quantities to the measurement models. These features need to be taken into account when evaluating uncertainty in measurements involving \( S \)-parameters. It has been shown that the GUM family of documents [2–6] has been an essential source of information for dealing with these features during the uncertainty evaluation process.
This article has focused on just some of these features: (i) how to represent the complex-valued $S$-parameters during an uncertainty evaluation process; (ii) the use of uncertainty matrices for expressing the uncertainty information; (iii) the use of correlation coefficients and uncertainty regions (i.e. ellipses) to aid in the representation and interpretation of the uncertainty information; (iv) how to use multiple complex-valued $S$-parameters to determine other measurement quantities; and (v) how to transform between the various data formats (e.g. impedance quantities) that are often used in conjunction with the $S$-parameters.

There are, however, topics relating to $S$-parameter uncertainty that have not been covered by this short review paper. These include the representation of $S$-parameter uncertainty in the vector magnitude and phase format. This is the format that is most commonly used by practising engineers and scientists. Some information on this subject has been given in [13]. In addition, the use of alternative regions of uncertainty (other than the ellipse) for complex-valued measurands has not been covered in this paper. Some information on this subject has been given in [14]. Although the use of a Monte Carlo method for propagating uncertainty has been referred to in this paper, more detailed information on using such techniques for evaluating uncertainty in $S$-parameter measurements can be found in [15]. Finally, the ‘frequency domain’ nature of the $S$-parameter measurements has not been discussed in this paper. This relates to the very common practise of measuring $S$-parameters sequentially at multiple (often, many hundreds of) frequencies. Such (large) data sets can be used subsequently to transform the measurement...
information into the time-domain (e.g. by applying a discrete inverse Fourier transform to the data in the frequency-domain). Some information on the propagation of uncertainty for this measurement situation has been given in [16].

Finally, it is worth stating, as we reach the conclusion of this short article, that the work that has been undertaken to date (and continues to be undertaken) by the Joint Committee for Guides in Metrology (JCGM) Working Group 1 (WG1) [17] has made an enormous positive impact, globally, on the many sectors of science, engineering and technology that rely of the use of electromagnetic metrology as a source of information. These sectors are too numerous to list here, but include consumer electronics (computers and smartphones), security (scanners at airports, etc), global climate change monitoring (environmental sensing), scanners for medical diagnostics, and, next-generation technologies (such as RF nanotechnology, etc). It is therefore vitally important that activities such as the JCGM WG1 continue in the years to come, so that generic uncertainty practices remain appropriate and fit-for-purpose for future emerging applications and requirements.

Acknowledgments

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