Use of reduced aperture waveguide as a calculable standard for the verification of millimetre-wave vector network analyzers

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Abstract—In this paper, 3D electromagnetic simulation is used to predict the attenuation of a length of reduced aperture waveguide (RAW) and to estimate the uncertainty in the predicted attenuation due to imperfect dimensions and alignment of the waveguide. This allows the RAW to be used as a calculable standard for the verification of mm-wave waveguide vector network analyzers (VNAs). The VNA is verified if the attenuation of the RAW measured on the VNA agrees with the predicted attenuation to within the uncertainties.

Keywords—Measurement standards, Electromagnetic modeling, Millimeter wave measurements.

I. VERIFICATION OF MILLIMETRE-WAVE WAVEGUIDE VECTOR NETWORK ANALYZERS

In recent years, millimetre- and sub-millimetre-wave Vector Network Analyzers (VNAs) have become available that operate in the frequency range 100 to 1100 GHz (i.e. 0.1 to 1.1 THz) and that are fitted with rectangular waveguide test ports [1 - 4]. Table I lists seven standard waveguide sizes [5] that together cover this frequency range.

Typically, the design of the interface (or “flange”) used to connect and align two sections of waveguide is the same for all the waveguide sizes listed in Table I. In particular, the flange used with WM-250 waveguide operating at 1000 GHz is usually the same as that used with WM-2540 waveguide operating at 100 GHz despite the fact that the width and height dimensions of a WM-250 waveguide are ten times smaller than those of a WM-2540 waveguide. Fig 1 shows a typical waveguide flange used in the frequency range 100-1100 GHz.

To gain confidence in the operation of a calibrated VNA, the VNA should be verified i.e. objective evidence should be provided that the S-parameters indicated by the VNA for a Device Under Test (DUT) agree with the “true” values of the S-parameters of the DUT. If an estimate of the uncertainties in either the indicated or the true S-parameters (or both) are available then the indicated and true S-parameters should agree within the stated uncertainties [6].

Two classes of device with known S-parameters suitable for use in verifying a VNA are as follows:

- Calculable devices - devices with a relatively simple geometry whose S-parameters can be calculated from the dimensions of the device based on the laws of electromagnetics by means of either mathematical analysis or numerical simulation;
- Characterised devices - devices with S-parameters which are stable in time and repeatable and which have been measured by a reputable calibration laboratory.

Such devices are known as “verification standards”. Five currently used waveguide VNA verification standards are listed in Table II together with their classification as either “calculable” or “characterised”. At present there are not many calibration laboratories with the capability to measure...
attenuation at frequencies above 110 GHz so at these frequencies the use of the characterised verification standards listed in Table II i.e. fixed and stepped attenuators may be less straightforward. In this paper, a sixth type of device is proposed for use as a calculable waveguide VNA verification standard - namely a section of “reduced aperture waveguide” (RAW) - see Table III. This is a section of rectangular waveguide for which both the height and the width are less than those of the VNA test ports. If the RAW is chosen to be one of the standard waveguide sizes [5], it should be straightforward to obtain RAWs of different lengths. Also the commonality of flanges between different waveguide sizes, mentioned previously, will allow the RAW to be connected to and aligned with the VNA test ports which are of a different waveguide size to the RAW.

This paper will consider the use of a section of WM-1295 waveguide of nominal length 1.47 mm to verify a WM-2540 waveguide VNA operating from 75 to 110 GHz i.e. a W-band waveguide VNA. Fig. 2 shows that the WM-1295 waveguide section is operating below cut-off across the whole frequency band of the VNA and so forms an out of band reduced aperture waveguide (OBRAW) with respect to the VNA test ports. Another possibility, which is not discussed further in this paper, would be to use a section of WM-1651 waveguide to verify the WM-2540 waveguide VNA. As shown in Fig 2, such a waveguide section would be operating below cut-off in the lower part of the VNA frequency band (below 90.8 GHz) and would be propagating in the upper part of the band and so would constitute a (partially) in band reduced aperture waveguide (IBRAW) with respect to the VNA test ports.

### Table II. Currently Used Waveguide VNA Verification Standards

<table>
<thead>
<tr>
<th>Waveguide VNA verification standard</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform waveguide</td>
<td>Calculable</td>
</tr>
<tr>
<td>Reduced height waveguide (“Beatty standard”) [7]</td>
<td>Calculable</td>
</tr>
<tr>
<td>Cross-connected waveguide [8]</td>
<td>Calculable</td>
</tr>
<tr>
<td>Step attenuator</td>
<td>Characterised</td>
</tr>
</tbody>
</table>

### Table III. Proposed New Waveguide VNA Verification Standard

<table>
<thead>
<tr>
<th>Waveguide VNA verification standard</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced Aperture Waveguide (RAW)</td>
<td>Calculable</td>
</tr>
</tbody>
</table>

II. REDUCED APERTURE WAVEGUIDE (RAW) AS A CALCULABLE VNA VERIFICATION STANDARD

In order to obtain a calculable verification standard, the attenuation of the 1.47 mm long section of RAW is predicted using the 3D electromagnetic simulation software CST Microwave Studio (CST MWS) [9]. The CST MWS calculations are based on an ideal RAW connected to perfect VNA test ports. The ideal RAW is assumed to have aperture dimensions equal to the nominal dimensions for WM-1295, length equal to the measured length of the RAW section and to be perfectly aligned with the test ports. The perfect test ports are assumed to have aperture dimensions equal to the nominal dimensions for WM-2540. The waveguide dimensions and alignments assumed in the calculations are summarised in Table IV. Other assumptions made include that the waveguide is perfectly conducting and that the waveguide walls are perfectly smooth and perfectly square. The calculated attenuation for the ideal RAW is plotted in Fig 4 as the curve labelled 'simulated'. It varies with frequency from about 30 dB to about 10 dB. A longer RAW would have a larger attenuation and a shorter RAW would have a smaller attenuation. Hence several lengths of RAW could be used to verify the VNA for transmission measurements over its complete dynamic range of operation.

Imperfections in the RAW result in errors in the predicted attenuation. Since these imperfections are not known exactly but are subject to uncertainty, so is the predicted attenuation. The uncertainty in attenuation is estimated by following the guidelines given in [10]. The following imperfections in the RAW are considered in the uncertainty evaluation:

- deviation of the height of the RAW from nominal (Δa)
- deviation of the width of the RAW from nominal (Δb)

![Fig. 2. Cut-off frequencies of WM-1651 and WM-1295 waveguides in relation to the WM-2540 VNA operating band](image-url)
TABLE IV. ASSUMED WAVEGUIDE DIMENSIONS AND ALIGNMENTS IN THE CALCULATION OF RAW ATTENUATION

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal broad wall dimension of RAW</td>
<td>(a_b)</td>
<td>2.540 mm</td>
</tr>
<tr>
<td>Nominal narrow wall dimension of RAW</td>
<td>(b_n)</td>
<td>1.270 mm</td>
</tr>
<tr>
<td>Nominal broad wall dimension of RAW</td>
<td>(a)</td>
<td>1.295 mm</td>
</tr>
<tr>
<td>Nominal narrow wall dimension of RAW</td>
<td>(b)</td>
<td>0.6475 mm</td>
</tr>
<tr>
<td>Measured length of RAW</td>
<td>(l)</td>
<td>1.478 mm</td>
</tr>
</tbody>
</table>

Translational misalignment of the RAW with respect to the VNA test ports along the broad waveguide wall

\[ \Delta x \] 0 mm

Translational misalignment of the RAW with respect to the VNA test ports along the narrow waveguide wall

\[ \Delta y \] 0 mm

Rotational misalignment of the RAW with respect to the VNA test ports

\[ \Delta \theta \] 0°

- uncertainty in the measured length of the RAW (\(\Delta l\))
- translational misalignment of the RAW parallel to the broad waveguide wall (\(\Delta x\))
- translational misalignment of the RAW parallel to the narrow waveguide wall (\(\Delta y\))
- rotational misalignment of the RAW (\(\Delta \theta\))

The estimated worst case magnitudes of these imperfections for the 1.47 mm length of RAW are given in Table V. Estimates of \(\Delta a\) and \(\Delta b\) are derived from measurements of the broad and narrow wall dimensions of the apertures of the RAW made using a coordinate measuring machine (CMM) fitted with a 0.3 mm diameter ball tip micro-stylus. The dimensions are measured at different positions across each aperture and at different depths inside the aperture. \(\Delta a\) and \(\Delta b\) are taken to be the worst case measured deviations from nominal [11]. The length of the RAW was measured using a micrometer and \(\Delta l\) is based on the estimated uncertainty in the micrometer measurements. \(\Delta x\), \(\Delta y\), and \(\Delta \theta\) are estimated as the maximum possible misalignments calculated from the dimensions and tolerances of the flange alignment pins and holes given in [12]. In practice, the flanges used in the frequency range 100-1100 GHz are more precise than those specified in [12] and so \(\Delta x\), \(\Delta y\), and \(\Delta \theta\) are probably over-estimated.

It is assumed that the predicted attenuation of the RAW, \(A\), at a particular frequency, is given by

\[ A = A_0 + \sum_{i=1}^{6} E_i \]

where \(A_0\) is the calculated attenuation of the ideal RAW and \(E_i\) is the error in attenuation due to the \(i\)th imperfection in the RAW (see the list of imperfections in Table V). It is further assumed that the errors \(E_i\) have nominal values equal to zero but are subject to uncertainty. Hence, assuming the uncertainty in the calculated attenuation of the ideal RAW, \(A_0\), is negligible; the combined standard uncertainty in the predicted attenuation of the RAW, \(u(A)\), is given by

\[ u(A) = \sqrt{\sum_{i=1}^{6} u^2(E_i)} \]

where \(u^2(E_i)\) is the square of the standard uncertainty in \(E_i\).

TABLE V. IMPERFECTIONS IN DIMENSIONS AND ALIGNMENT OF THE RAW

<table>
<thead>
<tr>
<th>Imperfection</th>
<th>Description</th>
<th>Symbol</th>
<th>Range of values ((J_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Deviation from nominal</td>
<td>(\Delta a)</td>
<td>± 0.014 mm</td>
</tr>
<tr>
<td>2</td>
<td>Deviation from nominal</td>
<td>(\Delta b)</td>
<td>± 0.004 mm</td>
</tr>
<tr>
<td>3</td>
<td>Deviation from measured</td>
<td>(\Delta l)</td>
<td>± 0.003 mm</td>
</tr>
<tr>
<td>4</td>
<td>Deviation from perfect</td>
<td>(\Delta x)</td>
<td>± 0.005 mm</td>
</tr>
<tr>
<td>5</td>
<td>Deviation from perfect</td>
<td>(\Delta y)</td>
<td>± 0.005 mm</td>
</tr>
<tr>
<td>6</td>
<td>Deviation from perfect</td>
<td>(\Delta \theta)</td>
<td>± 0.73°</td>
</tr>
</tbody>
</table>

The standard uncertainty, \(u(E_i)\), in the error in attenuation (expressed in dB) due to the \(i\)th imperfection in the RAW is estimated at a particular frequency as follows:

- An estimate is made of the range of possible values of imperfection \(i\) – this defines an interval \(J_i\) about zero that is independent of frequency - (see Table V);
- By means of electromagnetic simulation (using CST MWS), the corresponding range of possible values of the error in attenuation (expressed in dB) due to imperfection \(i\), \(E_i\), is estimated at the frequency of interest – this defines an interval \(I_i\);
- A rectangular distribution is assigned to the error in attenuation \(E_i\) at the frequency of interest with the width of the distribution being equal to the length of the interval \(I_i\), i.e. equal to \(L(I_i)\); \(u(E_i)\), is taken to be the standard deviation of this rectangular distribution i.e.

\[ u(E_i) = \frac{L(I_i)}{2\sqrt{3}} \]

Fig 3 show the relative sizes of the standard uncertainties in attenuation due to the six imperfections in the RAW listed in Table V. It can be seen that by far the largest cause of uncertainty for the particular RAW section being considered is the deviation of the broad wall dimension of the RAW from its nominal value. In fact, compared to this, the other imperfections make a negligible contribution to the overall uncertainty in the attenuation.

Finally, the expanded uncertainty in the predicted attenuation of the RAW, \(U(A)\), corresponding to a coverage
probability of approximately 95% is obtained by multiplying the combined standard uncertainty, \( u(A) \), by a coverage factor \( k = 2 \) to obtain 

\[ U(A) = 2u(A). \]

![Graph showing error (dB) vs. frequency (GHz)](image)

**Fig. 3.** Components of uncertainty for reduced aperture waveguide

### III. VERIFICATION OF A VNA USING A REDUCED APERTURE WAVEGUIDE (RAW)

A W-band waveguide VNA fitted with WM-2540 test ports was calibrated from 75 GHz to 110 GHz using a Thru Reflect Line (TRL) calibration. After calibration, the 1.47 mm length RAW was connected between the two test ports of the VNA and its voltage transmission coefficient (VTC) was measured. The magnitude of the measured VTC expressed in decibels (i.e. the measured attenuation) is plotted as a function of frequency in Fig 4 together with the estimated attenuation of the RAW derived from electromagnetic simulation and the expanded uncertainty interval in the predicted attenuation shown with dashed lines in the figure. It can be seen that the measured and predicted attenuation of the RAW agree within the uncertainties and so the VNA is verified for transmission measurements in this part of its dynamic range.

![Graph showing comparison of measured and simulated attenuation for reduced aperture waveguide with error bounds on the simulation (dashed curves)](image)

**Fig. 4.** Comparison of measured and simulated attenuation for reduced aperture waveguide with error bounds on the simulation (dashed curves)

### IV. CONCLUSIONS

This paper has demonstrated the use of a reduced aperture waveguide (RAW) as a calculable verification standard for transmission measurements made on waveguide VNAs. It has been shown how electromagnetic simulation can provide both a prediction of the attenuation of the RAW and an uncertainty in that prediction. Use of different lengths of RAW would provide a range of attenuation values and so would allow the VNA to be verified throughout its dynamic range of operation. Although illustrated here for W-band, RAW verification standards are also applicable in other mm-wave and sub mm-wave waveguide bands.

### REFERENCES


